CIS 1600 Recitation 12
Binomial and Geometric Distribution, Hall’s Theorem, Relations

November 17, 2023
Hall’s Theorem

Let $G = (X, Y, E)$ be a bipartite graph. For any set $S$ of vertices, let $N_G(S)$ be the set of vertices adjacent to vertices in $S$.

G contains a matching that saturates every vertex in $X$ iff $|N_G(S)| \geq |S|$, $\forall S \subseteq X$. (Hall’s condition)
A binary relation is a set of ordered pairs.

For example, $R = \{(1, 2), (2, 3), (5, 4)\}$

$(1, 2) \in R$: 1 is related to 2 by relation $R$, we denote this by $1R2$.

A binary relation $R$ from set $A$ to $B$ is a subset of the Cartesian product $A \times B$. 

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Relations
Properties of Relation

▶ Reflexive: for all \( x \in A \), \((x, x) \in R\).

▶ Irreflexive: for all \( x \in A \), \((x, x) \notin R\).

▶ Symmetric: for all \( x, y \in A \), \((x, y) \in R \implies (y, x) \in R\).

▶ Antisymmetric: for all \( x, y \in A \), \((x, y) \in R \) and \((y, x) \in R \implies x = y\).

▶ Transitive: for all \( x, y, z \in A \), \((x, y) \in R \) and \((y, z) \in R \implies (x, z) \in R\).

▶ Symmetric and antisymmetric are not opposites.