CIS 1600 Recitation 11
Binomial and Geometric Distribution, Relations, Memoryless Property, Chebyshev’s Inequality

November 9-10, 2023
Binomial Distribution

▶ A sequence of $n$ Bernoulli trials that are independent and each has a probability $p$ of success. How many successful outcomes?

▶ Example: A sequence of $n$ coin flips in which the probability of obtaining heads is $p$. How many flips result in head?

▶ A binomial r.v. $X$ with parameters $n$ and $p$ has the following distribution for $j = 0, 1, 2, ..., n$: 

$$Pr[X = j] = \binom{n}{j} p^j (1 - p)^{n-j}$$

▶ $E[X] = np$ and $Var[X] = np(1 - p)$
A sequence of Bernoulli trials that are independent with each having a probability $p$ of success, that stops after the first success.

Example: A sequence of coin flips in which the probability of obtaining heads is $p$. How many flips until we reach our first head?

$\Omega = \{H, TH, TTH, TTTTH, \ldots\}$

For any $\omega \in \Omega$ of length $i$, $Pr[\omega] = (1 - p)^{i-1}p$. 
A geometric r.v. \( X \) with parameter \( p \) has the following distribution for \( i = 1, 2, \ldots \)

\[
Pr[X = i] = (1 - p)^{i-1}p
\]

\( E[X] = \frac{1}{p} \) and \( Var[X] = \frac{1-p}{p^2} \)

**Memoryless Property.** For geometric r.v. \( X \) with parameter \( p \) and for \( n > 0 \) and \( k \geq 0 \),

\[
Pr[X = n + k \mid X > k] = Pr[X = n]
\]
Chebyshev’s Inequality

Let $X$ be a random variable. For all $a > 0$:

$$Pr\left[|X - E[X]| \geq a\right] \leq \frac{Var[X]}{a^2}$$
A binary relation is a set of ordered pairs.

For example, $R = \{(1, 2), (2, 3), (5, 4)\}$

$(1, 2) \in R$: 1 is related to 2 by relation $R$, we denote this by $1R2$.

A binary relation $R$ from set $A$ to $B$ is a subset of the Cartesian product $A \times B$. 