CIS 1600 Recitation 10
Eulerian & Hamiltonian Graphs, Tail Inequalities, Variance

November 2 - 3, 2023
Markov’s Inequality

Let $X$ be a non-negative random variable. For all $a > 0$:

Markov’s Inequality

$$Pr[X \geq a] \leq \frac{E[X]}{a}$$
Variance

- How much a random variable deviates from its mean.

- The variance of a random variable $X$ is defined as

$$\text{Var}[X] = E[(X - E[X])^2] = E[X^2] - (E[X])^2$$

- The standard deviation of a random variable $X$ is

$$\sigma[X] = \sqrt{\text{Var}[X]}$$

- If $X$ and $Y$ are independent random variables, then

$$\text{Var}[X + Y] = \text{Var}[X] + \text{Var}[Y]$$

and

$$E[XY] = E[X]E[Y]$$

(Note the converse of this statement is not true).
An Eulerian circuit of a graph $G$ is a closed walk in which each edge of $G$ appears exactly once.

A connected graph is Eulerian if it contains an Eulerian circuit.

A connected graph $G$ is Eulerian iff every vertex has even degree.

A Hamiltonian cycle in a graph $G$ is a cycle in which each vertex of $G$ appears exactly once.

A graph is Hamiltonian if it contains a Hamiltonian cycle.
Graph coloring

- A graph is \textit{k-colorable} if each vertex can be colored using one of the \( k \) colors so that adjacent vertices are colored using different colors.

- The \textit{chromatic number} of a graph \( G \), \( \chi(G) \), is the smallest value of \( k \) for which \( G \) is \( k \)-colorable.

- A \textit{bipartite} graph is a graph that is 2-colorable.

- A graph with maximum degree at most \( k \) is \((k + 1)\)-colorable.