This homework is due electronically on Gradescope at 11:59PM EDT, October 25, 2023. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. **\LaTeX**: Please typeset all your answers in \LaTeX{} based on the template we provide for you. Failure to do so will result in a 0 for the homework.

B. **Standard Deductions:**
   - 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. **Solutions**: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. **Collaboration**: Please make sure to strictly follow our collaboration policy as clarified on Piazza.

E. **Citations**: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. **Outside Resources**: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. **Late Policy**: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two ‘T’ homeworks and two ‘H’ homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.
1. [12 pts] Candy Coin Conundrum

Laura and Luna still haven’t figured out what costumes to wear on Halloween, and without any costumes, they won’t be able to go trick-or-treating! Luckily, Ishaan has offered them an alternative way to get candy this year. He has invented a new game with the following rules:

Before the game starts, Ishaan tosses a fair coin (one side of the coin is labeled 1 and the other side is labeled with a 2) and rolls a fair 160-sided die (the sides are labeled from 1 to 160), but he doesn’t show Laura or Luna the results.

Now, Laura and Luna must pay 20 candies to play. They then decide if they want to see the result of the coin toss or the die roll first. If the result of the object they choose is a 1 (with either the coin toss or die roll), they win 160 candies and move on to the next round; if it is any other number, they must pay Ishaan 32 candies and do not move on to the next round.

In the second round, Ishaan reveals the result of the other object. For instance, if Laura and Luna chose the die in the first round, they will now see the result of the coin toss, and vice versa. If the result of the second object is a 1, they win 1600 candies. If it is any other number, they lose and must pay Ishaan 32 candies.

Laura and Luna aren’t sure whether they want to play the game or not and want to strategize beforehand. Help Laura and Luna make an informed decision by answering the following questions. Note that you must provide justification and/or work to supplement your answer. Numeric values alone will not receive full credit.

(a) Let $X$ be a random variable that denotes the result of the coin flip. What is $E[X]$?

(b) Let $Y$ be a random variable that denotes the result of the die roll. What is $E[Y]$?

(c) If Laura and Luna decide to not play the game, what is the expected value of their candy winnings?

(d) If Laura and Luna decide to play the game and look at the coin first, what is the expected value of their candy winnings?

(e) If Laura and Luna decide to play the game and look at the die first, what is the expected value of their candy winnings?

2. [9 pts] Amazing Maize Maze May Delay TAs (from writing question names)

Rashmi loves Halloween and goes to a pumpkin patch with her friends to celebrate. They decide to go into the corn maze and somehow Rashmi (even with all of her genius intellect) can’t figure out how to get out of the maze. However, she notices that each time she turns, she will be one step closer to the end of the maze with probability $p$, and the probability that she has made no progress is $1 - p$. 

(a) What is the probability that after making two turns, Rashmi will be 1 step closer to escaping than she was when she started?

(b) What is the probability that after making three turns, Rashmi will be 2 steps closer to escaping than she was when she started?

(c) Given that after making three turns, Rashmi has moved 2 steps closer to escaping, what is the probability that after her first turn, Rashmi ended up 1 step closer to the end of the maze?

3. [9 pts] Saurabh’s Soccer Snack Scheme

Saurabh is on a journey to collect as much candy as he can so that he can snack on it while destroying all the other TAs during Friday soccer. On his journey, he notices that there are $2k$ houses and $k$ footpaths between them. Assume that $k$ is a positive integer, and that footpaths are bidirectional. Prove that if there are no isolated houses, then there are exactly $k$ groups of houses where each house in a group is reachable from all other houses in that group.