This homework is due electronically on Gradescope at 11:59PM EDT, October 23, 2023. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. \LaTeX: All solutions are required to be typeset in \LaTeX.

B. Standard Deductions:
   - 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. Collaboration: Please make sure to strictly follow our collaboration policy as clarified on Piazza.

E. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two ‘T’ homeworks and two ‘H’ homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.
0. Complete the Collaboration Quiz on Gradescope.

1. [20 pts] hi-BEAR-nation

Darebear, Saurabear, Clairebear and Karebear - the Elite Four - have all been collecting jars of honey. They’ve each collected 1600 jars in preparation for hibernation. To keep track of them, each person labels their jars from 1 to 1600. Darebear and Saurabear have Clover Honey jars, and Clairebear and Karebear have Manuka Honey jars (we can distinguish these species of honey).

So that they can tell their jars apart, Darebear and Clairebear put all of their jars into one backpack, and Saurabear and Karebear put all of their jars into another one. Now, we pick one jar uniformly at random from each backpack.

(a) Given that at least one of the two jars selected is a Clover Honey jar, what is the probability that both jars chosen are Clover Honey jars?

(b) Given that at least one of the two jars selected is a Clover Honey jar labeled with 37, what is the probability that both jars are Clover Honey jars?

2. [12 pts] hi-OCTOPUS-nation

Sar-bear is having a honey eating contest with her two friends Aar-Bear and Richard-Bear. Unfortunately, after many honey pots have been eaten, Sar-Bear and Richard-Bear have reached a tie. Being a CIS 1600 TA, Aar-Bear decides to break the tie by solving a probability question. Aar-Bear has both Sar-Bear and Richard-Bear choose a number from $[1..2n]$, where $n$ is a positive integer. We call the number that Sar-Bear picked $a$, and we call the number that Richard-Bear picked $b$. Aar-Bear defines events $X = \text{"a + b is even"}$, $Y = \text{"a \cdot b is even."}$ Aar-Bear then asks the following question: Are $X$ and $Y$ independent?

Help Sar-Bear and Richard-Bear answer Aar-Bear’s question and break the tie!

3. [24 pts] hi-GUINEA PIG-nation

The Elite Four are back! To keep their honey collecting skills in tip-top shape, they decide to host a friendly tournament amongst each other. Darebear declares himself the judge, leaving Saurabear, Clairebear, and Karebear as the participants.

Karebear proposes the following tournament structure. Karebear comes into the tournament as the first seed, which gives her the following advantage: Saurabear and Clairebear will compete against other twice to begin the tournament. If one of them is able to defeat the other both times, they will move on to face Karebear in the finals. They then have two chances to defeat Karebear, and must defeat her twice in a row in order to win the tournament. Note that if Karebear wins round 3 of the tournament, there will be no fourth game played. If Saurabear and Clairebear split their two game series, then Karebear will automatically be crowned the
champion.

As the judge, Darebear wants to analyze the fairness of the Karebear’s proposed structure. Based on prior performance, he’s aware of the following:

- The probability that Saurabear defeats Clairebear in a single match is 0.7.
- The probability that Karebear defeats Saurabear in a single match is 0.5.
- The probability that Karebear defeats Clairebear in a single match is 0.6.

Assume that all matches are independent and that it’s impossible to tie.

(a) Given that no matches have been played yet, determine the probabilities that:
   i. Karebear will win the tournament without playing a single match.
   ii. Saurabear will play Karebear in the finals.
   iii. Karebear is crowned the champion of the entire tournament.

(b) Given that Karebear played at least one match, determine the probabilities that:
   i. Saurabear is playing Karebear in the finals.
   ii. Karebear wins the overall tournament.

(c) Given that Karebear played exactly one match, what is the conditional probability that Saurabear defeated Clairebear in both of their matches?

4. [14 pts] hi-FLAMINGO-nation

Harish Bear-asubramanian is the mayor of a town of bears living in homes. To promote more friendship amongst the bears, he created roads between their homes! Between some pairs of homes, there is a navigable bidirectional road, with at most one road between any two homes. Harish wants to make two neighborhoods by partitioning all the homes into two non-empty sets: a set $S$ containing homes of neighborhood 1, and a set $W$ containing homes of neighborhood 2. He notices that no matter how he partitions the homes, there is always a road between some pair of homes $a$ and $b$ such that $a \in S$ and $b \in W$. Prove that this happens if and only if Harish can travel along some roads to get from any home to any other home in the whole town.