This homework is due electronically on Gradescope at 11:59PM EDT, October 9, 2023. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. **\LaTeX**: All solutions are required to be typeset in \LaTeX.  

B. **Standard Deductions:**
   - 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. **Solutions**: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. **Collaboration**: You may not collaborate with anyone via any means.

E. **Citations**: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. **Outside Resources**: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. **Late Policy**: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two ‘T’ homeworks and two ‘H’ homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.
1. **[12 pts] Confectionery Creature Coercively Carries Out Craving**

A group of 6 TAs just got back from Insomnia and each got 4 cookies. Oh no! They hear the footsteps of Ethan the Cookie Monster echoing throughout Houston Hall. Luckily, Ethan the Cookie Monster is feeling merciful today and will only eat 8 of their 24 total (distinguishable) cookies.

Ethan is a fair and just Cookie Monster: he will randomly choose the 8 cookies that he will consume. Each cookie is equally likely to be eaten. What is the probability that Ethan the Cookie Monster will eat at least one cookie from each TA?

2. **[10 pts] P Stands for Pumpkin, PIE, and PHP**

Luna, in addition to being an excellent CIS 1600 TA, is an exemplary baker who specializes in CIS 1600 themed cookies. She has decided to make Helen her guinea pig when testing recipes and Helen, who loves eating cookies, happily obliges. Luna will select one of three types of cookies and give them to Helen as a way to test her recipes. Luna has $a$ Pigeonhole Chestnut Praline cookies, $b$ Pumpkin Principle of Inclusion and Exclusion (P.I.E) cookies, and $c$ Chocolate Counting cookies for each of the next two days. She will choose a cookie uniformly at random for each day, and it’s possible Luna will give Helen the same type of cookie on both days. What is the probability that Luna will give Helen two different types of cookies. You may assume that $a + b + c \geq 2$.

3. **[14 pts] Hot, Hot, Hot, Ouch my Hands!**

Ria is preparing for the grand opening of her new shop, Insom-Ria Cookies! On the day of her grand opening, she decides she will toss 9 cookies at her first four customers at random. Each cookie is equally likely to land in any of the customers’ hands, and Ria never misses so we are sure every cookie will land in one of the customer’s hands.

However, Ria wants to make sure that none of her first four customers leaves empty handed. Therefore, if any of her first four customers do not get any cookies, Ria will toss 9 cookies again. What is the probability that Ria will only have to toss cookies once?

4. **[10 pts] Oh Oh Oh, O-Ria-s and Milan-Ohs**

Eric had some cookies that he was excited to feast on after a hard night of grading, but he came back home to see that all his cookies were stolen. Originally, he had some integer number of cookies between 0 and 8. Each cookie is distinct, but was one of two forms, either O-Ria-s or Milan-Ohs. One day, Nathan was able to find two of the lost O-Ria-s in the depths of a glass of milk. Eric tells Nathan that the chance that any two of his lost cookies, uniformly chosen at random, both being O-Ria-s, is exactly $1/2$. How many cookies did Eric lose, and how many of each type?
5. [12 pts] Unique Technique Leak
Suzzy the baker has discovered a new way to make the best chocolate chip cookies ever. Her new technique maximizes the number of chocolate chips in each cookie, and she wants to share her technique with the other bakers. At her next baking class, she counts that there are \( n \geq 2 \) people in attendance, including herself.

Ishaan, the tallest baker, claims that each of the \( n \) bakers in the meeting is friends with at least \( \frac{n}{2} \) of the other bakers. Note that each of these friendships is mutual (that is, if Baker A is friends with Baker B, then Baker B is friends with Baker A). Suzzy then tells all of her friends about the new technique; these friends tell all of their friends, and so on. Prove that all \( n \) bakers will eventually learn Suzzy’s technique.

6. [12 pts] (Lactose-free) Milk and Cookies
A group of diagnosed insomniac 160 TAs get into a kerfuffle on a midnight grading snack break (at Insomnia Cookies, of course). The 9 TAs break into two groups: 5 TAs who require milk with their cookies (Team Milk), and 4 lactose intolerant TAs who want nothing to do with dairy products (Team Anti-Milk). In an effort to better balance the groups and broker world peace, Team Milk trades one member to Team Anti-Milk, and Team Anti-Milk does the same. This results in two non-homogeneous groups: Team Milk having 4 lactose tolerant and 1 lactose intolerant TA, and Team Anti-Milk with 3 lactose intolerant and 1 lactose tolerant TA.

Finding the midnight squabble far more exciting than his glass of skim milk (he’s dieting in anticipation of national cookie day in December), the Milkman™ arrives and chooses one member from Team Anti-Milk, putting them on Team Milk. He then randomly selects a member of Team Milk, and finds them to be lactose intolerant. What is the probability that the Team Anti-Milk member transferred by the Milkman™ to Team Milk was lactose intolerant?