This homework is due electronically on Gradescope at 11:59PM EDT, September 20, 2023. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. **\LaTeX**: Please typeset all your answers in \LaTeX based on the template we provide for you. Failure to do so will result in a 0 for the homework.

B. **Standard Deductions**:  
   - 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. **Solutions**: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. **Collaboration**: You may not collaborate with anyone via any means.

E. **Citations**: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. **Outside Resources**: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. **Late Policy**: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two ‘T’ homeworks and two ‘H’ homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.
1. [9 pts] Outfit Oddity

A mysterious new outfit design was found to have interesting mathematical properties. In particular, it has been hypothesized that the below inequality holds true for any outfit of this design with height $n > 1$.

Prove that for all integers $n > 1$,

$$1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{n^2} < 2 - \frac{1}{n}$$

2. [12 pts] Michael the Merch Master

Being the prankster he is, Michael is trying to sneak onto a runway at New York Fashion Week wearing nothing but CIS 1600 Merch. Unfortunately, he is caught by a security guard before he can get to the runway. However, the guard is a good sport and is willing to let Michael onto the runway if he can prove the equality of two specific expressions. Help Michael out by giving a combinatorial proof for the following identity (other kinds of proofs will receive no credit).

**Note:** You should not algebraically manipulate the expressions; i.e. your answers to your counting question should be equal to the expressions on the LHS and RHS of the equality as they are given, not different algebraic expressions, even if they may be equivalent.

$$\sum_{i=0}^{r} \binom{i + m - 1}{m - 1} = \binom{m + r}{m}$$

where $m \geq 1$ and $r \geq 0$.

3. [9 pts] Catching Fashion Cache Using Cash

Rashmi, Elisa, and Harish are the most fashionable TAs on staff. As such, they decided to take the week off to go see the New York Fashion Week. Unfortunately, seeing their favorite fashion icons is expensive. For the group, the trip will cost $1200. Rashmi, Elisa, and Harish have each saved $1000 in $20 bills. They decide that each of them will begin by contributing $100 in $20 bills. (That is, everyone contributes at least $100 to the purchase.) After each of them have contributed their $100, each of them will contribute some amount of additional money to reach the required $1200 to go to Fashion Week. $m \in \{\text{Rashmi, Elisa, Harish}\}$, we denote $a_m$ by the number of $20 bills that TA $m$ contributes to the Fashion Week trip after they contributed 100 (the original $100 does not go towards this value). Count the number of ways Rashmi, Elisa, and Harish can contribute the remaining money to go on the trip, given that they can only contribute with $20 bills.