This homework is due electronically on Gradescope at 11:59PM EDT, September 25, 2023. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. **LATEX**: All solutions are required to be typeset in LATEX.

B. **Standard Deductions**:  
   • 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. **Solutions**: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. **Collaboration**: You may not collaborate with anyone via any means.

E. **Citations**: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. **Outside Resources**: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. **Late Policy**: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two ‘T’ homeworks and two ‘H’ homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.
1. **[14 pts] Jason Assembles Supreme Outfit Nomination**

It's New York Fashion Week and Jason is trying to determine which outfits from the runway shows are the best outfits. He devises a system to determine the best outfits, which he calls Supreme Outfits. There will be \( n \geq 1 \) outfits, and each outfit will go up against every other outfit in a one-on-one comparison with one clear winner and one clear loser (no draws or stalemates can occur!).

Note that victories are not transitive – outfit \( a \) beating outfit \( b \) and outfit \( b \) beating outfit \( c \) does not imply that \( a \) will have beat \( c \), as well.

Jason sets the following criteria for an outfit to be a Supreme Outfit: an outfit \( x \) is a Supreme Outfit if for all other outfits \( y \), either \( x \) beats \( y \) or \( x \) beats some third outfit \( z \) who beat \( y \).

Help Jason determine the best outfits and prove that at least one of the \( n \) outfits will be a Supreme Outfit.

2. **[10 pts] But Where’s Bella?**

Gigi Harish is planning to walk the runway for New York Fashion Week, and she does not know which dress to wear. She is planning to wear 3 different dresses during the week, out of the 40 total dresses that her designer, Versa-Chen, is offering her. Dresses are made out of up to only 4 colors - red, orange, blue, and purple. 25 of them have red, 30 of them have orange, 33 of them have blue, and 35 of them have purple. Being the colorful person she is, Gigi Harish wants to pick 3 dresses that all have all four colors. Prove that there exist at least 3 dresses such that each of the dresses has all 4 of the possible colors.

3. **[10 pts] Fashion Photography for the CIS 1600 Fans**

For New York Fashion Week, the CIS 1600 TAs host their own fashion show and each TA walks on the runway with their own outfit. All \( n \geq 2 \) TAs participate in the show. At the end of the show, the TAs take photos with each other to post on Instagram. Each photo only has 2 TAs in it. Any TA can take a photo with any number of other TAs (one at a time), including taking no photos at all (meaning they are in none of the photos). Additionally, two TAs can only take a photo together once.

Prove or disprove the claim that there must always be at least two TAs who took the same number of photos.

4. **[14 pts] Life of the Party**

After watching modern luxury designer Phillip Lim’s denim runway show, Ria gets in line for a star-studded NYFW after-party. She gets moved to the front of the queue because of her impeccable sense of style, but must now complete a (combinatorial) proof of identity if she wishes to gain entry to the exclusive event. Help Ria by giving a combinatorial proof to show
that for all integers $n \geq 3$,

$$3^{n-2} \cdot n \cdot (n - 1) = \sum_{k=2}^{n} \binom{n}{k} (k)(k - 1)2^{k-2}$$

5. [10 pts] Darren’s Daring Designer Diagram
Darren feels his drip has been lacking recently and wants to create new outfits from clothing featured at New York Fashion Week. He remembers seeing Helmut Lang, Coach, and Dion Lee at the fashion week and decides that he will choose articles of clothing from these three designers. Darren decides to use a $(x, y)$-coordinate plane to plan out the perfect outfit. He sets out a 100-by-100 grid and he arranges the articles of clothing such that at each point $(a, b)$ where $a, b \in \mathbb{Z}$ on the plane, there is exactly one article of clothing from one of the 3 designers. He notices that regardless of the arrangement he chooses, he can always create a rectangle in which the designer at each corner are the same. Prove that Darren’s observation is true!

6. [12 pts] Friendly Fashion
Winston Chen-el, the drippiest of the CIS 1600 TAs, is holding a fashion show to highlight some of his favorite fits. The fashion show takes place over $n \in \mathbb{Z}^+$ days. To avoid any sort of audience pressure, Winston Chen-el only invites his friends to attend. Prove by induction that the total number of friends who attend the fashion show is at most the sum of the number of friends attending each day. Note that for a friend to attend the show they must be present on at least one of the $n$ days. Please make sure to first phrase the claim using mathematical notation, and only proofs (using mathematical notation) by induction will receive credit.