This homework is due electronically on Gradescope at 11:59PM EDT, September 06, 2023. To receive full credit all your answers should be carefully justified. Additionally, make sure to fill out the Gradescope Policy Quiz!

Please make note of the following:

A. **\LaTeX**: Normally, we require all solutions to be typeset in \LaTeX. We have provided a \LaTeX primer video on Piazza and on the course website under the ‘resources’ tab, and have provided a template, should you choose to use \LaTeX. However, \LaTeX is not strictly required for this first assignment only.

B. **Standard Deductions:**
   - 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. **Solutions:** Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. **Collaboration:** You may not collaborate with anyone via any means.

E. **Citations:** All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. **Outside Resources:** Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. **Late Policy:** We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two ‘T’ homeworks and two ‘H’ homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.
1. [9 pts] Open Sesame?
Darren is having the time of his life at NSO. He pulls up to the ∃∀N frat house but is stopped at the door by Ishaan and Andrew. They refuse to let Darren into the house unless he is able to answer their CIS 1600 questions. Help Darren with the following proofs so he can show off his pong skills.

(a) Let $A = \{n \in \mathbb{N} \mid n = 7k + 4, \text{ for some } k \in \mathbb{N}\}$ and $B = \{n \in \mathbb{N} \mid n = 21k + 4, \text{ for some } k \in \mathbb{N}\}$. Prove that $B \subset A$.

(b) Let $A = \{n \in \mathbb{Z} \mid n = 4 - k, \text{ for some } k \in \mathbb{N} \text{ and } 5|k\} \text{ and } C = \{m \in \mathbb{Z} \mid m = 24 - 5k \text{ for some } k \in \mathbb{N} \text{ and } k \geq 4\}$. Prove that $A = C$.

(c) Let $A = \{n \in \mathbb{N} \mid n = 6k - 13, \text{ for some } k \in \mathbb{N}\}$ and $B = \{m \in \mathbb{N} \mid m = 5k + 14, \text{ for some } k \in \mathbb{Z}^+\}$. Prove that $A \neq B$.

2. [6 pts] Post Coffee Penn Card Placed Carelessly
Darren walks to the Quad with 10 heavy suitcases after a refreshing morning coffee that helped him recover from all that partying the night before. Unfortunately though, he’s left his Penn Card at Stommons and does not want to walk back with all his suitcases! The Quad Office tells Darren that he can enter without one if he could give a proof for $\sqrt{6} + \sqrt{7} < \sqrt{26}$. Darren responds instantly with the following:

Squaring both sides of $\sqrt{6} + \sqrt{7} < \sqrt{26}$ gives $13 + 2\sqrt{42} < 26$, which further implies $2\sqrt{42} < 13$. Squaring both sides gives $168 < 169$, which is true.

Help the Quad Office verify Darren’s solution. If it is valid, give a brief justification why (a couple of lines will suffice). If not, explain why the proof is invalid and provide a correct proof.

3. [8 pts] Gatekeep >: ( Rashmi needs move-in carts, and fast! She goes to Hill College House to find some, but to her horror, discovers that the carts have all been rounded up and gatekept by CIS majors. In order for Rashmi to take out any carts, the CIS majors tell her that she must first prove or disprove each of the following statements:

(a) For every prime $p$, either $p + 10$ is a prime or $p + 11$ is a prime.

(b) $\forall x, y \in \mathbb{R}^+, |x + y| = |x| + |y|$.

(c) $\forall x \in \mathbb{N}, (x + 1)^3 - 5x^2 + 6$ is odd.

(d) $\forall q, r, x, y, s \in \mathbb{Z}, (s|q \land s|r) \implies s|(qx + ry)$.

(e) $\forall i, j, k \in \mathbb{Z}$, if $i - j$ is even and $j - k$ is odd, then $i - k$ is odd.

Help Rashmi prove or disprove each of these statements so that she can continue her move-in!
4. [7 pts] **Houston Hall Haul**

Thomas just arrived at Penn for NSO and wants to find out about all of the delicious food at Penn! He decides to go to Houston Hall, where there are now $n$ distinct places to grab food. Thomas decides he will either spend swipes or dining dollars at each place, but not both. He may also skip a place if it looks unappetizing to him at the moment, but he wants to be sure he ends up with at least one item at the end (so he doesn’t start skipping meals his first week!) In how many ways can Thomas order food?