This homework is due electronically on Gradescope at 11:59PM EDT, September 11, 2023. To receive full credit all your answers should be carefully justified. **Additionally, make sure to fill out the Gradescope Policy Quiz!**

Please make note of the following:

A. **\LaTeX**: Normally, we require all solutions to be typeset in \LaTeX. We have provided a \LaTeX primer video on Piazza and on the course website under the ‘resources’ tab, and have provided a template, should you choose to use \LaTeX. However, \LaTeX is not strictly required for this first assignment only.

B. **Standard Deductions:**
   - 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. **Solutions:** Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. **Collaboration:** You may not collaborate with anyone via any means.

E. **Citations:** All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. **Outside Resources:** Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. **Late Policy:** We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two ‘T’ homeworks and two ‘H’ homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.
1. **[8 pts] Hill College House Survival Guide**

   Freshman year, Sana had heard too many horror stories of food poisoning and dining halls, so she decides to go try each of them one by one on her own. She chooses to visit Hill’s dining hall first, where she notices that the room is in the shape of a cube with integer side lengths. She makes an observation that “if the volume of the dining hall is an even integer, the side length of the room would be too!” Can you help Sana prove her claim? Recall that the volume of a cube with side length $s$ is $s^3$.

2. **[8 pts] Victor’s Transportation Realization**

   Victor is part of a group of 864 distinct freshmen who are running late to the gala at the Philadelphia Museum of Art. Traffic is extremely bad, so they know they will miss the entire event if they travel by car or bus. Luckily, Victor happens to be an avid biker with a lot of bikes. He has 432 bicycles, so to get everyone to the gala, two freshmen will need to ride on each bike. How many ways can Victor pair up the 864 freshmen to create partners for each bike? (Note that the order of the freshmen within a pair does not matter, and that the order of the pairs doesn’t matter either).

3. **[12 pts] Freshmen Flyer Frenzy**

   Sam is running a SAC fair booth for his new club, and to convince the 3 freshmen approaching his table to join, he decides to give them flyers. By pure coincidence, Sara happens to be selling a bulk deal of 8 (distinguishable) fancy flyers in different colors. Sam decides to purchase all 8 of Sara’s flyers for the 3 (distinguishable) freshmen. He wants to give at least one flyer to each of the 3 freshmen. How many ways can Sam give away his 8 flyers to the 3 freshmen? Note that all flyers must be given away to a freshman.

4. **[8 pts] This’ll Make an Awesome Dorm Decoration**

   Karen is decorating her new room with pictures of all her memories with her PHINS group. She initially arranges the 256 pictures in a $16 \times 16$ grid and decides she wants to frame her favorite ones. Each frame can only be given to one picture and all the pictures will stay in their position on the grid.

   (a) Karen is very picky, and she decides that she wants to distribute 16 indistinguishable frames to her pictures such that no two framed pictures are in the same row or column. How many ways can Karen frame the pictures?

   (b) Karen realizes that she wants to spice it up a bit more by making the frames different colors. Now Karen has 16 different colored distinguishable frames. How many ways can she distribute the distinguishable frames such that no two framed pictures are in the same row or column?
5. [12 pts] Everything Reminds Me of The Quad
From a tiny window in her sad sophomore housing, Alex gazed down at a bunch of eager freshmen moving into the Quad. She felt an extra wave of nostalgia and reminisced about her own NSO experience. Naturally, she wanted to pay tribute to her first year so she waltzed down Spruce street towards something more familiar. She lined up for a chicken bowl, in true Quad-fashion. Mr. McClellander, working behind the counter, is in a playful mood and refuses to give her extra spicy mayo unless she helps him with a proof! As you know, it is not the full culinary experience without extra spicy mayo - there is a lot on the line, folks.

Let $x_1, x_2, \ldots, x_{2021}$ be a permutation of the numbers from 1 to 2021. Prove or disprove that the product

$$(x_1 - 1)(x_2 - 2) \cdots (x_{2020} - 2020)(x_{2021} - 2021)$$

must be an even number to help Alex and her taste buds relive freshman year.

Suzzy, the freshman, is moving into her dorm room in the Quad. Due to construction, there is no light in her room. So, Suzzy buys $n$ ($n \in \mathbb{N}$ and $n \geq 4$) lamps to ensure that her room is as bright as possible. She places lamps so that each lamp forms the corners of a convex polygon with $n$ sides. Recall that a convex shape is one where all diagonals lie entirely within the polygon. Next, Suzzy hangs strings of fairy lights between every non-adjacent pair of lamps. Assuming that no more than two strings of fairy lights intersect at the same point, how many intersections do the strings of fairy lights have at interior points?

7. [11 pts] PHINS in PHILLY
This year, Harish, the PHINS leader, is trying something new with his PHINS group to increase mingling between the new frosh.

Nervous, the $2n$ distinct freshman each stood in pairs next to their NSO besties (NSO besties are a mutual pairing), where $n \in \mathbb{N}$, $n \geq 2$. After watching the freshman not talk to each other, Harish, a 1600 TA, decided to use combinatorics to increase intermingling. He arranges the students in a circle so that they could meet new people. But he needs help figuring out the number of possible arrangements.

Note that two arrangements are considered the same if, for each freshman, the freshman to his or her left is the same in both arrangements. Suppose that in the freshman circle, $k$ out of the $2n$ freshman where $k \leq n$ demand to be next to their NSO bestie. There is at most 1 demanding freshman per NSO bestie pair. The remaining freshman may or may not be next to their NSO bestie. How many ways are there to arrange the $2n$ freshman in a circle such that the $k$ demanding freshmen get to be next to their NSO bestie?