1. [10 pts] No Story No Name :( 
Give answers to the following questions. You do not have to show your work for this question.

(a) Write the following sets explicitly, i.e. list the members of these sets.
   i. \( \{ x \mid x \text{ is a square of an integer, } x \text{ is odd, and } x < 220 \} \)
   ii. \( \{ x \mid x \text{ is an integer such that } x^2 = 47 \} \)
   iii. \( \{ x \mid x \text{ is a real number such that } x^2 - 160 = 9 \} \)
   iv. \( \{ x \mid 3x \text{ is a positive integer less than 57 and } 7|x \} \)

(b) Use the set builder notation to give a nontrivial description of each of these sets (nontrivial means that your solution should not simply enumerate every element).
   i. \( \{ 8, 11, 16, 22, 24, 32, 33, 40, 44 \} \)
   ii. \( \{ 4, 19, 44, 79, 124 \} \)
   iii. \( \{ -4, -3, -2, -1, 0, 1, 2, 3, 4, 5 \} \)

(c) What is the cardinality of each of the following sets?
   i. \( \{ a, \{\{a}\} \} \)
   ii. \( \{\{p, m\}, \emptyset\} \)
   iii. \( \{d, \{d\}, \{d, \{d\}\}\} \)

(d) Determine whether each of the following is true or false.
   i. \( \emptyset \subseteq \{ x \} \)
   ii. \( \emptyset \in \{ x \} \)
   iii. \( x \subseteq \{ x \} \)
iv. \( \{x\} \subseteq \{x\} \)

v. \( \{x\} \subseteq \{x\} \)

vi. \( \{x\} \subseteq \{\{x\}\} \)

(e) What is the power set of \( \{x, y, z\} \), where \( x, y, \) and \( z \) are distinct elements? What is the cardinality of the powerset?

(f) Find two sets \( A \) and \( B \) such that \( A \in B \) and \( A \subseteq B \).

2. [8 pts] Play Chance Relay to Allay Vacay Delay

Tired of being CIS 1600 TAs, Karen, Yijia, Saurabh, Hasit, and Ria are on an adventure to Barbie Land in hopes of avoiding the upcoming grading session. Before they’re allowed to enter, a Barbie immigration officer lines them up in alphabetical order and tells them the following: the first two people must flip a coin to decide who gets to enter Barbie Land first. The winner of the match can then enter, while the loser must stay to play the next person in line. The person who wins the next match gets to enter second, while the loser must again stay to play the next person in line. This process continues until there is only 1 TA left, who enters last. In how many orders can the 5 TAs enter Barbie Land?

3. [10 pts] Fashion Forward Physicists

Winnie Dong-enheimer recently hired \( n \) new physicists to join the Manhattan Project, where \( n \) is a positive integer. Before they can start working, Winnie Dong-enheimer decides that it is of paramount importance that these new physicists are first dripped out, so she decides to dye their lab coats with 118 different colors (one for each element on the periodic table!). To decide how to color her physicists, she makes them line up next to each other in a row. She decides that she’d like the colors to be symmetric about the middle of the \( n \) physicists. For example, if \( n = 4 \), then Winnie Dong-enheimer might color her physicists Red-Yellow-Yellow-Red; if \( n = 5 \), she could choose Orange-Blue-Green-Blue-Orange. Unfortunately, Winnie Dong-enheimer has more important issues to attend to, so she needs your help to do the calculations. How many different ways are there to color the row of \( n \) physicists with 118 colors such that the colors are symmetric about the middle?


Winnie Dong-enheimer needs more uranium ore ASAP! She orders two of her newly hired physicists, Michael and Megan, to get some for her at the local uranium factory. When they arrive, they find a row of 200 bags of uranium ore ready to go. The bags are of varying weights, and since each bag rests on a scale, both Michael and Megan know the exact weight of each bag. They decide that they will each transport 100 bags back to Los Alamos. Feeling competitive and wanting to one-up each other in hopes of gaining Winnie Dong-enheimer’s approval, they
each want to bring back the heavier load, where the weight of a load is the sum of the weights of the 100 bags that make up the load. They choose bags in the following way: Michael removes a bag from either end of the original row and adds it to his load. Then Megan removes a bag from either end of the remaining row of 199 bags. They continue alternating like this until there are no more bags remaining.

Michael wants to select bags so that in the end, the total weight of his 100 bags is at least as heavy as the total weight of Megan’s 100 bags. Can you help him find a strategy? If no strategy exists, explain why.

5. [8 pts] Pink and Purple Presents
Barbie Rashmi loves the CIS 1600 staff! She decides to bring each member of the CIS 1600 staff a present. The presents are either Pinkalicious Pink or Lovely Lavender, and there’s at least one of each. Barbie Rashmi wants everyone to feel special on staff, so she makes the presents distinguishable. Barbie Rashmi arranges the presents in a circle and for each present, she wants the neighboring presents to be a different color than itself. If Barbie Rashmi uses 76 Pinkalicious Pink presents in her circle, how many Lovely Lavender presents are there in her circle?

6. [10 pts] I’m a Barbie Girl in a Barbie Dream Dollhouse
Thomas Li-eynman has been working with Winnie Dong-enheimer in Los Alamos for quite some time, and he’s starting to miss his old house. Although Thomas Li-eynman may not be able to transport his old house to Los Alamos, he can do the next best thing: Get a Barbie Dream Dollhouse, of course! One day, while walking around the town, he stumbles across a vast collection of Barbie Dream Dollhouses. 15 of the dollhouses are Pink, 7 of the dollhouses are Purple, and 11 of the dollhouses are Red. All houses of the same color are indistinguishable from each other. Eager to show the other physicists in Los Alamos, Thomas Li-eynman decides to bring some of the dollhouses back to his friends. He wants to do this in such a way that he always brings back at least one dollhouse. For instance, he might bring back 0 Pink dollhouses, 2 Purple dollhouses, and 3 Red dollhouses, but he may not bring back 0 dollhouses of all three colors. He is strong enough to carry all 33 dollhouses back to his friends. How many different ways can Thomas Li-eynman bring the dollhouses back to his physicist friends?

7. [12 pts] Not So Suspicious Barbie Transaction
Oppen-erren has decided to stop being a physicist and start being a Barbie Doll Seller. Oppen-erren’s Barbie Doll Manufacturer, Nathan Ken, promises to deliver him exactly 508 barbies at 2:00 PM on Sunday, September 3rd. Oppen-erren is planning to start selling these Barbies at 2:00 PM on Monday, September 4th. However, Nathan Ken is a bit salty that the world sees him as Just (Nathan) Ken, so he decides to sneak a Weird Barbie in with the set of 508. Whoever plays with the Weird Barbie will lose their fingers and toes at exactly 1:59 PM on Monday.
Oppen-erren, wanting to prevent the Weird Barbie from being sold, hires nine Barbie-Testers who would be willing to lose their fingers and toes for Oppen-erren.

How can Oppen-erren organize his Barbie-Testers to identify the Weird Barbie before Oppen-erren starts selling his Barbies? Justify your answer.