This homework is due electronically on Gradescope at 11:59PM EDT, November 20, 2023. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. \LaTeX: All solutions are required to be typeset in \LaTeX.

B. Standard Deductions:
   • 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. Collaboration: Please make sure to strictly follow our collaboration policy as clarified on Piazza.

E. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two ‘T’ homeworks and two ‘H’ homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.
1. **[30 pts] The Very Best Baked Beans**
Michael is planning on hosting Thanksgiving dinner for the rest of the 1600 staff. However, he’s never cooked for so many people before and is unsure how many dishes to prepare. Michael decides to pick uppercase letters uniformly at random with replacement, with each letter corresponding to a different dish (A for apple pie, B for bread pudding, C for cranberry sauce, etc.). For each letter picked, Michael will make one serving of its corresponding dish. He will continue picking letters until he picks an M directly followed by a T (when this occurs, he will make the turkey corresponding to the final T and then stop). Michael is unsure whether this procedure is likely to result in too many or few servings of dishes being made. What is the expected number of servings of dishes that Michael will prepare?

Hint: Try applying the memoryless property.

2. **[20 pts] Peas are Beans, Sometimes**
Thanksgiving is right around the corner. The CIS 1600 staff are planning to have a Thanksgiving feast. To decide what food will be served at the feast (turkey or chicken), Hasit will ask each of the TAs whether they are on Team Turkey or Team Chicken. Each of the \( n \) TAs is either on Team Turkey with probability \( p \) or team Chicken with probability \( 1 - p \), and every TA joins exactly 1 team. Let \( A \) denote the total number of TAs on Team Turkey and let \( B \) the total number of TAs on team Chicken. Help Hasit solve the following problems!

(a) Determine the probability mass distribution for random variable \( A \).

(b) Determine the expected value of \( A \).

(c) Determine the variance of \( A \).

(d) What is the probability that the first TA to choose ends up being the only person in that team?

(e) What is the probability that at least one team ends up with a total of exactly one person?

(f) What is the expectation and the variance of the difference \( D = A - B \)?

3. **[20 pts] Soybean Security**
Nathan has been arranging pies and trails of cranberry sauce for the 1600 staff Thanksgiving Feast. Each cranberry sauce trail runs bidirectionally between exactly two pies.

For the arrangement of pies and cranberry trails \( (P, C) \), he defines the following types of subsets

- **Independent Set** – A set of pies \( S \subseteq P \), such that for any two distinct pies \( u, v \in S \), \( \{u, v\} \notin C \).

- **Clique** – A set of pies \( S \subseteq P \), such that for any two distinct pies \( u, v \in S \), \( \{u, v\} \in C \).
Further, they denote:

- $\alpha(G)$ as the size of the maximum independent set in $G$
- $\kappa(G)$ as the size of the maximum clique in $G$
- $\chi(G)$ as the minimum number of flavors of pies Nathan would need to have in $G$ such that any two pies that share a cranberry trail are of different flavors. (Two pies that are directly connected by a single cranberry trail must not be the same flavor)

Nathan is pretty sure there are some relationships between the symbols they came up with, but he’s having trouble figuring out what they are specifically. For any arrangement of pies and cranberry trails, help Nathan find the **strictest** relationship that holds between the following quantities (less than, less than or equal to, equal to, greater than or equal to, greater than, not related). Remember to prove your answers.

(a) $\alpha(G)$ and $\kappa(\overline{G})$.

(b) $\chi(G)$ and $\kappa(G)$. 