This homework is due electronically on Gradescope at 11:59PM EDT, November 8, 2023. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. \texttt{\LaTeX}: Please typeset all your answers in \LaTeX based on the template we provide for you. Failure to do so will result in a 0 for the homework.

B. Standard Deductions:
   • 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. Collaboration: Please make sure to strictly follow our collaboration policy as clarified on Piazza.

E. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two ‘T’ homeworks and two ‘H’ homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.
[0 pts] An Announcement:
Please complete the Mid-Semester Feedback Form on Canvas.

1. [10 pts] beans

BEANS BEANS BEANS! The CIS 1600 staff loves beans and strings! Andrew, being the biggest
bean fan, decides that he will start to only communicate using beans. That is, he can send strings
made of beans by printing a stream of uppercase letters uniformly at random from the alphabet
onto each bean. Andrew wants to be able to express his love through beans with beans by
constructing the string "WELOVEBEANSALLBEANSYUM" through this stream of lettered
beans. What is the fewest number of random bean letters Andrew would need to enter for the
expected number of occurrences of "WELOVEBEANSALLBEANSYUM" to be at least 1?

2. [12 pts] It’s Bean a Lifetime

There’s one thing that Sid loves most in the world: beans! Lucky for him, he received a special
offer in the mail to win a lifetime supply of beans. However, there’s one catch. He must solve
the following problem:

Let \( Y \) be a random variable such that \( Y = \sum_{i=1}^{n} Y_i \), where each \( Y_i \) is a random variable.
Prove that if \( E[Y_iY_j] = E[Y_i]E[Y_j] \), for every pair \( i, j \), such that \( 1 \leq i < j \leq n \), then
\[
Var[Y] = \sum_{i=1}^{n} Var[Y_i].
\]

Unfortunately Sid has been daydreaming about beans during lecture and isn’t sure what the
answer is. Help Sid win the lifetime supply of beans by solving the problem for him!

3. [8 pts] Bowls and Bowls of Beans

Shaurya is a very picky eater when it comes to his burrito bowls! Each bowl has two choices
for beans: black beans and pinto beans. Shaurya can distinguish between a black bean and a
pinto bean, but he can’t tell any pair of black beans apart, and he can’t tell any pair of pinto
beans apart either.

Shaurya orders a burrito bowl with \( N \) total beans, of which exactly \( r \) are black (and the rest
are pinto). If he randomly takes a spoonful of \( b \) beans from the bowl independently and without
replacement, what is the expected number of pinto beans he selects?