This homework is due electronically on Gradescope at 11:59PM EDT, November 13, 2023. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. \LaTeX: All solutions are required to be typeset in \LaTeX.

B. Standard Deductions:
   - 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. Collaboration: Please make sure to strictly follow our collaboration policy as clarified on Piazza.

E. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two ‘T’ homeworks and two ‘H’ homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.
1. [14 pts] The Very Best Baked Beans

Dilini loves a good old fashioned full English Breakfast, but the best part is always the baked beans! Hasit is a baked bean aficionado and promises to cook her the best baked beans she’s ever had, but only if Dilini can prove or disprove the following conjecture about graphs:

Let $G$ be a simple, undirected graph with at least one edge. Construct another graph $G'$ as follows — for each edge $e$ in $G$, we create a unique corresponding vertex $v_e$ in $G'$. Then, for any two vertices $v_{e_1}$ and $v_{e_2}$ in $G'$, there is a corresponding edge $\{v_{e_1}, v_{e_2}\}$ in $G'$ if the edges $e_1$ and $e_2$ in $G$ are incident on the same vertex. If $G$ has an Eulerian circuit, then $G'$ has a Hamiltonian cycle.

Help Dilini prove or disprove the statement so she can complete her English breakfast with steaming hot baked beans!

2. [10 pts] Peas are Beans, Sometimes

Thomas is attending the Black Eyed Peas concert, but unfortunately his phone is dead and he does not have his ticket! Each ticket corresponds to a number, and each ticket has a different number of digits in its number, so Thomas needs to make sure he has the right number of digits for his ticket number. Let Thomas’s ticket number be represented by $z$.

He knows any ticket number $z$ starts with 1 and has digits appended to the end of it, each chosen mutually independently and uniformly at random from $[0..9]$. This process is repeated until $z$ is even or $z$ has 6 digits. Help him figure out the expected value of the number of digits in $z$ at the end of this process so he can enter the concert in time!

For example, one possible instance of this process would be

$$1 \rightarrow 17 \rightarrow 173 \rightarrow 1731 \rightarrow 17310,$$

at which point the process stops because $z = 17310$ is even.

3. [10 pts] Soybean Security

Soy-zy Wang has an insane obsession with soybeans and will be distraught if anyone were to steal any from her. Thus, she decides to put them in a vault and needs to generate a 12-character password by selecting each character independently and uniformly at random from $\{a, b, \ldots, z\} \cup \{A, B, \ldots, Z\} \cup \{0, 1, \ldots, 9\}$ to ensure maximum security.

(a) What is the probability that exactly 6 of the characters are digits?

(b) What is the expected number of digits in a password?

(c) What is the variance of the number of digits in a password?
4. [12 pts] Bean Farming Beans

Sara Xin-to’Beans is creating a connected graph to map out all of the bean farms in Pennsylvania. 2 farms have a bidirectional edge between them if there is a dirt road between them. Additionally, Sara sorts all of the farms by the types of beans they produce: Xin-to beans, Luna beans and Canne-Dilini beans (these are terrible bean puns so feel free to suggest your own to us). These differences are represented in the graph by different colorings assigned to each node. Therefore, we can say that this graph of farms $G$ is a connected graph with 3 or more vertices, and that $\chi(G) = 3$. Help Sara prove the following claim on the graph:

Consider a proper 3-coloring of $G$ with farms producing Xin-to beans, Luna beans and Canne-Dilini beans. Prove that there exists a Xin-to bean farm that has both a Luna bean neighbor and a Canne-Dilini bean neighbor.

5. [12 pts] Disclose Bean Preferences Beforehand!

Selina is buying lunch for a bunch of the 1600 TA’s at Chipotle. Upon getting there, she realizes she forgot to poll everyone for their bean preference, and decides to randomly allocate beans to everyone’s burritos. The Chipotle has 30 servings of pinto beans available and 30 servings of black beans available, and Selina wants to put all of these into 30 separate burritos. Bean servings are assigned to a burrito at random, meaning that one burrito may have as little as 0 or as many as 60 servings of beans. She hopes that she will end up with a good mix of pinto and black beans in each burrito so everyone will have at least some of the beans of their true preference. Help her out by calculating the expected number of burritos with at least one serving of each type of beans.

6. [12 pts] Bean Stonks

Saurabh and Michael are the bestest of friends – one could even say they are Bean-FFs! However, like all friends, they sometimes have bean-f with one another. The source of their current dispute is as follows. Saurabh has some arbitrary random variable $X$, and Michael has some arbitrary random variable $Y$ such that $X$ and $Y$ are independent. Saurabh asserts that $X^2$ and $Y^2$ must also be independent, while Michael disagrees and says that it depends on what $X$ and $Y$ are. Unfortunately for the bean-boozeled Michael, Saurabh is correct. Help convince Michael that this is indeed the case by proving Saurabh’s assertion!