This homework is due electronically on Gradescope at 11:59PM EDT, November 1, 2023. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. **LaTeX**: Please typeset all your answers in LaTeX based on the template we provide for you. Failure to do so will result in a 0 for the homework.

B. **Standard Deductions**:
   - 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. **Solutions**: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. **Collaboration**: Please make sure to strictly follow our collaboration policy as clarified on Piazza.

E. **Citations**: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. **Outside Resources**: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. **Late Policy**: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two ‘T’ homeworks and two ‘H’ homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.
1. [12 pts] Cloudy with a Chance of Meatballs
Shaurya goes to Commons, where they are serving spaghetti and meatballs. As usual, the food is terrible, so instead of eating, he begins playing with his food. Shaurya creates a network of meatballs and noodles $G = (V, E)$, where every meatball $v \in V$ is connected to other meatballs via noodles, and each noodle $e \in E$ is between two meatballs and there is at most one noodle between each pair of meatballs. Megan and Laura then arrive at Commons and see Shaurya’s network. They decide to create connected noodle networks using the same set of meatballs as Shaurya’s network, but since Commons is starting to run out of food, they want to minimize the number of noodles needed. As a result, they each create distinct and minimally-connected networks of noodles and meatballs. In other words, each of their networks is a spanning tree of $G$.

Megan’s network consists of the spanning tree $T = (V, E)$, and Laura’s network consists of the spanning tree $T' = (V, E')$. This gains Shaurya’s attention and he notices that for any noodle $e$ such that $e \in E \setminus E'$, there is a noodle $e' \in E' \setminus E$ such that $(T - e) + e'$ and $(T' + e) - e'$ are also valid spanning tree networks. Prove that Shaurya is correct.

2. [9 pts] May the Best Pasta Win
Sara and Saurabh are both pasta enthusiasts. Recently, they got in a heated debate about what type of pasta is best. Sara thinks spaghetti is best, while Saurabh thinks fettuccine reigns supreme. To settle the debate, Sara proposes a game. The game consists of creating a $3 \times 3$ grid of fair coins. Each coin is tossed independently and comes up either heads (H) or tails (T). Sara needs to bet on the probability of having at least one $2 \times 2$ square of heads (H’s) in the grid to beat Saurabh. Help Sara find the probability of this event.

3. [9 pts] Pasta and Probability, Our Two Favorite Things
Winston Fettu-Chen-1 and Alex Ravi-Oh-li are pasta fiends, and order pasta every week during grading. There was a mess up at the restaurant this week, and only one plate of pasta was delivered. After debating for a while, neither of them is willing to give up this plate of delicious pasta to the other. Fortunately, Elisa has a plan: To determine who gets the pasta, Elisa states that both of them must predict a number from 1-6 inclusive. She then rolls 4 fair 6-sided dice. The person who correctly predicts the largest integer that appears on any of the four dice will be declared the winner. Compute the expected value of the largest integer that appears on any of the four dice, and tell your favorite TA so they will have a better chance of munching down that pasta!