

CIS 160 Recitation #9

Trees and Expectation

Trees

A **spanning tree** is a spanning subgraph that is a tree.

Every connected graph contains a spanning tree.

A **rooted tree** is a tree in which one vertex is distinguished from the others and is called the root.

A **binary tree** is a rooted tree in which every internal vertex has at most two children.

A **full binary tree** is a rooted tree in which every internal vertex has exactly two children.

Expectation

$$\mathbf{E}[X] = \sum_i ip_X(i) = \sum_i i \Pr[X = i]$$

$$\mathbf{E}[cX] = c\mathbf{E}[X]$$

Linearity of Expectation:

$$\mathbf{E} \left[\sum_{i=1}^n X_i \right] = \sum_{i=1}^n \mathbf{E}[X_i]$$

Indicator Random Variables

We can define an indicator random variable X_i such that:

$$X_i = \begin{cases} 1 & \text{desired outcome occurs} \\ 0 & \text{otherwise} \end{cases}$$

These are useful to break up problems into smaller components. For an example, look at the birthday problem from Thursday's lecture.