

## Recitation Guide - Week 9

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**Topics Covered:** Expectation, Linearity of Expectation, Spanning Trees

**Problem 1:**

Taki says to Yuyang, “Let’s play a game. I first roll a fair 6-sided die. If the number that shows up is divisible by 3, I roll again and I pay you the dollar amount that shows up on the second roll. If not, then I flip a fair coin. If it is tails, I take 10 dollars from you, and if it is heads, I pay you 5 dollars. What is your expected payoff?”

Yuyang, who does not want to lose to Taki again, has asked you to help him out. Should he play the game?

**Problem 2:**

There are  $n$  people in a room. Each pair of people has probability  $p$  of being friends (uniform probability across all pairs of people). What is the expected number of friend groups of size  $m$  in the room (in terms of  $n$ ,  $p$ , and  $m$ )? Friend groups are groups of people in which everyone in the group is friends with everyone else in the group. Note that a person can be in more than one friend group.

**Problem 3:** Consider a connected graph  $G = (V, E)$  and an arbitrary partition of  $G$ 's vertex set  $V$  into nonempty sets  $S$  and  $V \setminus S$ . Prove that if there exists only one edge  $e$  between the vertices in  $S$  and the vertices in  $V \setminus S$ , then  $e$  must be in every spanning tree of  $G$ .