

Recitation Guide - Week 9

Topics Covered: Linearity of Expectation, Variance, Bipartite Graphs

Problem 1: Era, as a busy college student, hasn't done laundry in weeks. In particular, she realizes that she has no more socks to wear, so she goes to the laundry room and throws in her 2 distinct pairs of socks. However, the machine is broken, and so it only returns 2 of her socks at random! Note that the two socks in a pair are also distinguishable.

- (a) What is the expected number of pairs that she can wear now?
- (b) What if she throws in n pairs and only gets $k > 1$, socks back?

Problem 2:

A 10 digit number with no zeroes is chosen by independently and randomly selecting each digit (1 - 9). Let N be the number of digits missing from the 10 digit number. For example, if the number is 1231452832, then we are missing the digits 6, 7, 9 so $N = 3$. Find $\mathbb{E}[N]$ and $\text{Var}[N]$.

Problem 3:

Prove that a graph is bipartite if and only if it has no odd length cycles.