

CIS 160

## Recitation Guide - Week 8

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**Topics Covered:** Trees, Independence

**Problem 1:**

Let  $T$  be a tree where the maximum degree is  $\Delta$ . Prove that  $T$  has at least  $\Delta$  leaves.

**Problem 2:**

Suppose  $E$  and  $F$  are independent events and  $\Pr[E] > 0$ . Let  $\bar{F}$  denote the complement of  $F$ . Are  $E$  and  $\bar{F}$  independent? Prove your answer.

**Problem 3:**

We have three wooden buckets,  $T_A, T_B, T_C$  and we throw  $n \geq 3$  metal keys in them. The key throws are mutually independent and each key is equally likely to land in each of the three buckets.

- (a) Let  $A$  be the event that after all keys are thrown bucket  $T_A$  has at least one key in it and similarly associate an event  $B$  with  $T_B$ . Are  $A$  and  $B$  independent? Justify your answer.
- (b) Compute the probability that after all keys are thrown, each of the three buckets has at least one key in it. Justify your answer.