CIS 160

Recitation Guide - Week 8

Topics Covered: Expectation, Independence, Spanning Trees

Problem 1:

Taki says to Yuyang, “Let’s play a game. I first roll a fair 6-sided die. If the number that shows up is divisible by 3, I roll again and I pay you the dollar amount that shows up on the second roll. If not, then I flip a fair coin. If it is tails, I take 10 dollars from you, and if it is heads, I pay you 5 dollars. What is your expected payoff?”

Yuyang has asked you to help him out. Should he play the game?

Problem 2:
We have three wooden buckets, A, B, C and we throw $n \geq 3$ metal keys in them. The key throws are mutually independent and each key is equally likely to land in each of the three buckets.

(a) Let A be the event that after all keys are thrown, bucket A has at least one key in it and similarly associate an event B with B. Are A and B independent? Justify your answer.

(b) Compute the probability that after all keys are thrown, each of the three buckets has at least one key in it. Justify your answer.

Problem 3: Consider a connected graph $G = (V, E)$ and an arbitrary partition of G's vertex set V into nonempty sets S and $V \setminus S$. Define edges that cross the cut between S and $V \setminus S$ to have an endpoint in S and an endpoint in $V \setminus S$. Prove that if there exists only one edge e that crosses the cut S and $V \setminus S$, then e must be in every spanning tree of G.