CIS 160 Recitation 7

Conditional Probability, Total Probability Theorem, Trees

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Conditional Probability

- We are interested in event A given that we already know another event B has happened.
- $Pr[A|B]$ "The probability of A given B"

$$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]}$$

- When $Pr[B] = 0$, $Pr[A|B]$ is undefined

Total Probability Theorem

- Consider events E and F. Since F and $\bar{F}$ partition the sample space:

$$\begin{aligned} Pr[E] &= Pr[E \cap F] + Pr[E \cap \bar{F}] \\ &= Pr[F] \cdot Pr[E|F] + Pr[\bar{F}] \cdot Pr[E|\bar{F}] \end{aligned}$$

- In general, if $A_1, A_2, \ldots, A_k$ partition the sample space, then

$$\begin{aligned} Pr[E] &= \sum_{i=1}^{k} Pr[E \cap A_i] \\ &= \sum_{i=1}^{k} Pr[A_i] \cdot Pr[E|A_i] \end{aligned}$$

Trees

For an n-vertex graph G, the following are equivalent and characterize trees with n vertices.

1. G is a tree (in other words, connected and acyclic)
2. G is connected and has exactly $n - 1$ edges.
3. G is minimally connected, i.e., G is connected but $G - \{e\}$ is disconnected for every edge $e \in G$.
4. G is maximally acyclic, ie., contains no cycle but $G + \{x, y\}$ does, for any two non-adjacent vertices $x, y \in G$.
5. Any two vertices of G are linked by a unique path in G.

Spanning Trees

- A **spanning subgraph** of the graph $G = (V, E)$ is a subgraph whose vertex set is the entire set V
- A **spanning tree** of a connected graph G is a spanning subgraph that is a tree.
- Every connected graph has a spanning tree.