CIS 160

Recitation Guide - Week 5

Topics Covered: Strong Induction, Pigeonhole Principle

Problem 1: Suppose we have the following sequence:

$$a_1 = 1 \qquad\qquad a_2 = 3 \qquad\qquad a_i = a_{i-2} + 2a_{i-1}, \quad i \in \mathbb{Z}, i \geq 3$$

Use strong induction to prove that for all integers $n \geq 1$, a_n is odd.

Problem 2: Anusha and Brandon are playing a game in which there are two non-empty bags with an equal number of marbles in them. In this game, the two players take turns removing marbles from one of the bags. In each turn, the player can remove any positive number of marbles as long as they are all from the same bag. The winner of the game is the player that removes the last marble. In Anusha and Brandon's current configuration, both bags initially start with the same number of marbles. Prove that one of them can guarantee a win.

Problem 3:

Let S be a set of 16 distinct positive integers such that $\forall x \in S,\ x < 60$. Show that there exist distinct integers $a, b, c, d \in S$ such that $a + b = c + d$.