

CIS 160 Recitation 5

Binomial Theorem, Pigeonhole Principle, Strong Induction

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The Binomial Theorem

- ▶ The binomial theorem gives an expression for $(a + b)^n$
- ▶ For any real numbers a and b and non-negative integer n

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

The Pigeonhole Principle

- ▶ $k + 1$ or more objects are distributed among k bins
- ▶ There is at least one bin that has two or more objects.

Generalized Pigeonhole Principle:

- ▶ n objects are placed into k boxes
- ▶ There exists at least 1 box containing at least $\lceil \frac{n}{k} \rceil$ objects.

Strong Induction

- ▶ Let n_0 be a natural number and let $P(n)$ be a predicate for all natural numbers $n \geq n_0$.
- ▶ Base Case: $P(n_0)$ holds
- ▶ Induction Hypothesis: $P(j)$ is true for $n_0 \leq j \leq k$
- ▶ Induction Step: We want to show that $P(k+1)$ is true.
That is, for any $k \geq n_0$,
$$P(n_0) \wedge P(n_0 + 1) \wedge \dots \wedge P(k) \implies P(k + 1) \text{ is true.}$$
- ▶ Strong and ordinary induction are mathematically equivalent.