

# CIS 160 Recitation #4

Induction, Binomial Theorem, Pigeonhole Principle

# Induction Review

Let  $P(n)$  be a predicate whose truth depends on  $n$ .

We want to prove that  $P(n)$  is true for all integers  $n$  greater than or equal to some integer  $n_0$ .

Base Case: Show  $P(n_0)$  is true.

Induction Hypothesis: Assume  $P(k)$  is true for some integer  $k \geq n_0$

Induction Step: Using this assumption, prove that  $P(k + 1)$  is true.

# Binomial Theorem

For any real numbers  $a$  and  $b$  and non-negative integer  $n$

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

# Pigeonhole Principle

- If  $k + 1$  objects are distributed into  $k$  bins, there will be at least one bin with at least 2 objects
- Generalized PHP: If  $n$  objects are distributed into  $k$  bins, there will be at least one bin with at least  $\lceil \frac{n}{k} \rceil$  objects
- When using PHP, cite it and describe what your “pigeons” (objects) are and what your “holes” (bins) are
- Tip: often a good technique for problems asking you to prove existence (look for “there exists”)