CIS 160

Recitation Guide - Week 4

Topics Covered: Induction, Pigeonhole Principle

Problem 1:

All the sheep in Arnold's flock have the same color! Arnold claims that he can use induction to prove all sheep in the world have the same color. Find the fault in his reasoning.

Base Case: Size = 1. One sheep, one color. ✓

Induction Hypothesis: Assume that in a flock of size k, where $k \in \mathbb{Z}^+$, all sheep have the same color.

Induction Step: We want to prove the claim is true for a flock of size $k + 1$. Take one sheep, let's call it Dolly, out. What remains is a flock of size k, so by IH, they all share the same color. Now put Dolly back in and take out another sheep, let's call it Polly, out. By IH, what remains is a flock of size k, so by IH, they all share the same color. Dolly and Polly must share the same color as they both are the same color as the other $k - 1$ sheep. Thus we arrive at the conclusion that all $k + 1$ sheep share the same color.

Problem 2:

Prove using induction that for any positive integer n and for any integers $d_0, d_1, \ldots, d_{n-1} \in [0..9]$ we have:

$$\sum_{j=0}^{n-1} d_j \cdot 10^j < 10^n$$

Problem 3:

Let S be a set of 16 distinct positive integers such that $\forall x \in S,\ x < 60$. Show that there exist distinct integers $a, b, c, d \in S$ such that $a + b = c + d$.