

# CIS 160 Recitation 4

Multisets, Sticks and Crosses, Combinatorial Proofs

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# Multisets

An unordered collection where repetitions are allowed.

- ▶ Permutation of Multiset:  $n_k$  are of type  $a_k$  and indistinguishable from one another.

$$\{n_1 \cdot a_1, n_2 \cdot a_2, \dots, n_k \cdot a_k\}$$

- ▶ Number of Permutations:

$$\frac{n!}{n_1! n_2! \cdots n_k!}$$

Where  $n = n_1 + n_2 + \cdots + n_k$

# Sticks and Crosses

- ▶ The number of ways to distribute  $r$  indistinguishable objects into  $n$  distinguishable containers.
- ▶  $r$  crosses
- ▶  $n - 1$  sticks

$$\binom{n+r-1}{r} = \binom{n+r-1}{n-1} = \frac{(n+r-1)!}{(n-1)!r!}$$

# Combinatorial Proofs

Asked to prove an identity.

- ▶ Pose a counting question.
- ▶ Answer the question in one way, giving us the LHS.
- ▶ Answer the question in another way, giving us the RHS.
- ▶ Since the LHS and RHS answer the same question, they must be equal.
- ▶ No algebraic manipulation of the identity should be done throughout the proof.

# Proof Techniques: Recap

1. Direct Proof
2. Contradiction
3. Contrapositive
4. Mathematical Induction
5. Combinatorial Proofs