CIS 160

Recitation Guide - Week 4

Topics Covered: Induction, Sticks and Crosses, Combinatorial Proofs

Problem 1:

All the sheep in Tien's flock have the same color! Tien claims that he can use induction to prove all sheep in the world have the same color. Find the fault in his reasoning.

Base Case: size = 1. One sheep, one color. ✓

Induction Hypothesis: Assume that in a flock of size k, where $k \in \mathbb{Z}^+$, all sheep have the same color.

Induction Step: We want to prove the claim is true for a flock of size $k+1$. Take one sheep, let's call her Winnie, out. What remains is a flock of size k, so by IH, they all share the same color. Now put Winnie back in and take out another sheep, let's call him Nicky, out. By IH, what remains is a flock of size k, so by IH, they all share the same color. Winnie and Nicky must share the same color as they both are the same color as the other $k-1$ sheep. Thus we arrive at the conclusion that all $k+1$ sheep share the same color.

Problem 2:

A janitor needs to distribute soap bars and toilet paper to customers of the hotel. He starts his shift with 10 bars of soap and 10 rolls of toilet paper. After the 6th room, he discovers that he has run out of supplies. Most importantly, he does not remember when his supplies ran out (meaning he could have used all his supplies in the first room). He cannot tell the difference between any two toilet paper rolls and between any two soap bars. However, he can easily tell the difference between toilet paper and soap bars.

How many ways could he have distributed the toilet paper rolls and soap bars to the different rooms?

Problem 3:

Give a combinatorial proof for the following, where $m \leq n$:

$$\sum_{k=0}^{m} \binom{n}{k} \binom{n-k}{m-k} = 2^m \binom{n}{m}$$

Problem 4:

Prove using induction that for any positive integer n and for any integers $d_0, d_1, \ldots, d_{n-1} \in [0..9]$ we have:

$$\sum_{j=0}^{n-1} d_j \cdot 10^j < 10^n$$