CIS 160 Recitation #3

Induction, Multisets, and Combinatorial Proofs

Propositions (brief review)

Sample proposition: Prove that the product of a non-zero rational number and an irrational number is irrational.

Contrapositive: If the product of two numbers is rational, then the two numbers are not a non-zero rational number and an irrational number.

Contradiction: Assume for the sake of contradiction that the product of a non-zero rational number and an irrational number is rational.

Induction

Let $P(n)$ be a predicate whose truth depends on n.

We want to prove that $P(n)$ is true for all integers n greater than or equal to some integer n_0 .

Base Case: Show $P(n_0)$ is true.

Induction Hypothesis: Assume $P(k)$ is true for some integer $k \geq n_0$

Induction Step: Using this assumption, prove that $P(k+1)$ is true.

Multisets

If we have a multiset M with n_1 objects of type a_1, n_2 objects of type a_2, …, and n_k objects of type a_k, such that objects of the same type are indistinguishable from one another, then the number of permutations of the objects in M is:

$$\frac{(n_1 + n_2 + ... + n_k)!}{n_1!n_2!...n_k!}$$

Sticks and Crosses

What if we want to take r-combinations with repetition?

Think of “sticks” as dividers between categories of objects.

Think of “crosses” as objects that we assign to each category.

Then, if we have n categories and we want to select r objects with repetition, we “permute” $n - 1$ sticks and r crosses:

$$\binom{n+r-1}{r}$$ r-combinations

Combinatorial Proofs

- We can prove that two expressions are equivalent by showing that they are both a solution to the same counting question.
- To do so, we come up with a counting question and two valid counting procedures that answer the question, one that results in one expression, and another that results in the other expression.

- We will do an example today.