

Recitation Guide - Week 3

Topics Covered: Sticks and Crosses, Combinatorial Proof, Multiset

Problem 1:

A janitor needs to distribute soap bars and toilet paper to customers of the hotel. He starts his shift with 10 bars of soap and 10 rolls of toilet paper. After the 6th room, he discovers that he has run out of supplies. Most importantly, he does not remember when his supplies ran out (meaning he could have used all his supplies in the first room). He cannot tell the difference between any two toilet paper rolls and between any two soap bars. However, he can easily tell the difference between toilet paper and soap bars.

- (a) How many ways could he have distributed the toilet paper rolls and soap bars to the different rooms?
- (b) How many ways could he have distributed the toilet paper rolls and soap bars given that there is at least one toilet paper roll and one soap bar in each room?

Problem 2:

Give a combinatorial proof for the following, where $m \leq n$:

$$\sum_{k=0}^m \binom{n}{k} \binom{n-k}{m-k} = 2^m \binom{n}{m}$$

Problem 3 (if time allows):

How many 5 letter sequences can be made from the letters in the word "PIAZZA"?