1. Prove the following:

$$(A \cup B) \setminus C \overset{C}{=} (A \setminus C) \cup (B \setminus C)$$

$\boxed{1}$

Let x be an arb. but part. element in the LHS

$x \in (A \cup B) \setminus C$

① $x \in A \cup B$ ② $x \notin C$

Case I: $x \in A$ $x \notin C$

WTS $x \in (A \setminus C) \cup (B \setminus C)$

$x \in A \setminus C$

$x \in (A \setminus C) \cup (B \setminus C)$ ✓

Case II. $x \in B$ $x \notin C$

$x \in B \setminus C$

$x \in (A \setminus C) \cup (B \setminus C)$ ✓

Thus, we have shown LHS $\subseteq$ RHS

RHS $\subseteq$ LHS

$\boxed{2}$ $(A \setminus C) \cup (B \setminus C) \subseteq (A \cup B) \setminus C$

Let y be an arb. but part elem in RHS

$y \in (A \setminus C) \cup (B \setminus C)$

① ②

$\underbrace{y \in A \setminus C}$ or $\underbrace{y \in B \setminus C}$

Case I: $y \in A$ $y \notin C$

$y \in (A \cup B) \setminus C$

$y \in A \cup B$

$y \in (A \cup B) \setminus C$ ✓

Case II: $y \in B$ $y \notin C$

$y \in A \cup B$

$y \in (A \cup B) \setminus C$ ✓

Thus RHS $\subseteq$ LHS

Because LHS $\subseteq$ RHS and RHS $\subseteq$ LHS, LHS = RHS ✓

2. Given numbers 1-9, how many permutations of the number do not have at least 7 consecutively increasing numbers.

Note: 1, 2, 3 is consecutively increasing

1, 4, 6 is not

(1-7) (2-8) (3-9)

1. Pick 7 cons ↑ # 3 way
2. Order 7 cons, and 2 other #'s in a line 3!

$\underline{(1\text{-}7)}$ $\underline{8}$ $\underline{9}$ 1 2 3 . . . 8 9

$\underline{1}$ $\underline{2\text{-}8}$ $\underline{9}$ 1 2 3 . . . 8 9

A: 7 cons increasing #s starting at pos'n 1

B: " " " pos'n 2

C: " " " " pos'n 3

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PIE

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$6 + 6 + 6 - 2 - 1 - 2 + 1$$

$$= 14$$

$|A|$: ________ __ __

1. Pick 7 cons #s (3 ways)
2. Pick a # in 8^{th} pos'n (2 ways)
3. Pick a # in 9^{th} pos'n (1 ways)

By MR, $3 \times 2 \times 1 = \boxed{6 \text{ ways}}$

$|A| = |B| = |C| = 6$

$|A \cap B| =$ _ _ _ _ _ _ _ _ _

1. Pick 8 cons $\uparrow$ #s (2 ways)
2. Pick last digit (1 way)

$= 2$ ways

$|B \cap C| =$ _ _ _ _ _ _ _ _ _ $= 2$

$|A \cap C| =$ 1 2 3 4 5 6 7 8 9 $= 1$

$|A \cap B \cap C| = 1$

$9! - 14$

3. Prove that the product of a non-zero rational and irrational number is irrational.

Contradiction: product of a rational and irrational number is rational

a: rational b: irrational

WTP ab = rational

$a = \frac{p}{q}$ for integers p, q and $\boxed{q \neq 0}$

$ab = \frac{x}{y}$ " " x, y " $y \neq 0$

$$ab = \frac{x}{y}$$

$$\frac{q}{p} \cdot \frac{p}{q} \cdot b = \frac{x}{y} \cdot \frac{q}{p}$$

$$b = \frac{x}{y} \cdot \frac{q}{p}$$

$x \cdot q$ is an integer

$y \cdot p$ is an integer, non-zero

$y \neq 0$ $p \neq 0$

b is rational