CIS 160 Recitation 3

Permutations, Combinations, Inclusion-Exclusion, Mathematical Induction

September 16-17, 2021

Permutation of Selected Elements

- Consider permutations of r elements out of n elements (r-permutation).
- By MR, the number of r-permutations equals

$$\begin{aligned} P(n;r) &= n \quad (n \quad 1) \quad \vdots \quad (n \quad (r \quad 1)) \\ &= n \quad (n \quad 1) \quad \vdots \quad (n \quad r+1) \\ &= \frac{n!}{(n \quad r)!} \end{aligned}$$

The Inclusion-Exclusion Formula

- If $A; B; C$ are any finite sets,

$$jA \ [\ Bj = jAj + jBj \quad jA \setminus Bj$$
$$jA \ [\ B \ [\ Cj = jAj + jBj + jCj \quad jA \setminus Bj \quad jA \setminus Cj \quad jB \setminus Cj + jA \setminus B \setminus Cj$$

- If $A; B; C$ are mutually disjoint ($A \setminus B = A \setminus C = B \setminus C = ;$)

$$jA \ [\ Bj = jAj + jBj$$
$$jA \ [\ B \ [\ Cj = jAj + jBj + jCj$$

This is called *addition rule*.

Combinations

- r-combination of a set of n elements = an unordered selection of r of the n elements.
- "n choose r":

$$\binom{n}{r} = \frac{n!}{r!(n \quad r)!}$$