CIS 160 Recitation #2

Sets, Permutations, and Combinations

Set Operations

Let A and B be sets.

$A \cup B$ is the set containing elements that are in A, B, or both (union).

$A \cap B$ is the set containing elements that are in both A and B (intersection).

If $A \cap B = \emptyset$, then A and B are disjoint.

$A \setminus B$ is the set of elements of A that are not in B.

$\bar{A}$ is the complement of A (all elements in consideration minus those in A)

$A \times B$ is the cartesian product of A and B, or the set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$.

Sets (cont.)

$A \times B = B \times A$ iff $A = B$ and A, B are both non-empty or both empty.

DeMorgan's Laws:

$$A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$$

$$A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$$

Permutations of Selected Elements

$P(n, r)$ denotes the number of permutations of r objects from n.

$$P(n, r) = \frac{n!}{(n-r)!}$$

Principle of Inclusion-Exclusion (PIE)

If $A, B,$ and C are finite sets, then:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

More generally, if we have finite sets $A_1, A_2, \ldots, A_n$, then:

$$\left|\bigcup_{i=1}^{n} A_i\right| = \sum_{i=1}^{n} |A_i| - \sum_{\substack{i,j \\ i \neq j}} |A_i \cap A_j| + \sum_{\substack{i,j,k \\ i \neq j, j \neq k \\ i \neq k}} |A_i \cap A_j \cap A_k| - \cdots + (-1)^{n-1} |\cap_{i=1}^{n} A_i|$$

Combinations

What if we want to choose r objects from n, but ordering does not matter?

$$\binom{n}{r} = \frac{P(n,r)}{r!} = \frac{n!}{(n-r)!r!}$$