Weekend Recitation 9/11

Counting, Sets, Proof Techniques

weilinhu@wharton
Weilin Hu

OH: Sundays 12-2PM @Skirkanich 114

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Propositions

Let p, q be propositions.

- $\neg p$: T when p is F, F when p is T
- $p \wedge q$: T when both p and q are T, F o.w.
- $p \vee q$: T when one of p or q is T, F o.w.
- $p \oplus q$: T when exactly one of p or q is T, F o.w.
- $p \implies q$: T when p is T q is T or when p is false
- $p \iff q$: T when p is T q is T or p is F q is F

  $\equiv p \to q \wedge q \to p$

Proof Methods: Direct

- To prove that a claim p is true, you assume information directly in or extrapolated from the proposition.

- **Example.** m and n are integers. Prove that if $m + n$ is even, then $m - n$ must also be even.

Proof Methods: Contradiction

- To prove that a claim p is true, assume towards contradiction that $\neg p$ is true. Then, we have to show that $\neg p$ is false by reaching a contradiction (some statement that is always false).

- **Example.** If there are 25 people in a room, then at least 3 people share the same birthday month.

Proof Methods: Contrapositive

- To prove that $p \implies q$ is true, we can show that $\neg q \implies \neg p$ is true, since they are logically equivalent.

- **Example.** If x and y are integers where $x + y$ is even, then x and y are both odd or both even.

Sets

- A *set* is an unordered collection of distinct objects.
- Two set are *equal* if and only if they have the same elements.
- The *cardinality* $|S|$ is the number of distinct elements of S.
- A is a *subset* of B ($A \subseteq B$) if and only if every eleemnt of A is an element of B.
- A *powerset* of S ($P(S)$) is a set of all possible subsets of S.

Set Operations

- *Union:* $A \cup B$ contains elements either in A or B, or both.
- *Intersection:* $A \cap B$ contains element in both A and B.
- *Disjoint:* A, B are disjoint if $A \cap B$ is an empty set.
- *Partition:* $\{A_1, A_2, ..., A_n\}$ is a partition of A if and only if (i) $A = \cup_{i=1}^{n} A_i$ and (ii) $A_1, A_2, ..., A_n$ are mutually (pairwise) disjoint.
- *Difference:* $A \setminus B$ or $A - B$ contains elements that are in A but not in B.
- *Complement:* $\bar{A}$ contains all elements not in A.
- *Cartesian Product:* $A \times B = \{(a, b) | a \in A, b \in B\}$.

DeMorgan's Laws

- Let A, B, C be set.

$$A \setminus B \cup C = (A \setminus B) \cap (A \setminus C)$$

$$A \setminus B \cap C = (A \setminus B) \cup (A \setminus C)$$

Counting

- **Multiplication Rule.** If a procedure can be broken down into k steps and the first step can be performed in n_1 ways, the second step can be performed in n_2 ways, ..., the k^{th} step can be performed in n_k ways *regardless of how preceding steps were performed*, then the entire procedure can be performed in $n_1 \times n_2 \times .... \times n_k$ ways.
- **Permutations.** A permutation of a set of *distinct* objects is an ordering of the objects in a row. By MR, the number of permutations of n distinct objects is $n \times (n-1) \times (n-2) \times ... \times 2 \times 1 = n!$.