CIS 160

Recitation Guide - Week 2

Topics Covered: Proofs, Counting

Problem 1: Show that for any two integers m and n, $m^2 + n^2$ has the same parity as $m + n$.

Solution:

Consider the following two cases.

Case 1: $m + n$ is odd

We can write $m + n = 2k + 1$, for some $k \in \mathbb{Z}$. Then, we have,

$$\begin{aligned} m^2 + n^2 &= (m + n)^2 - 2mn \\ &= (2k + 1)^2 - 2mn \\ &= 4k^2 + 4k + 1 - 2mn \\ &= 2(2k^2 + 2k - mn) + 1 \end{aligned}$$

which is odd, as $2k^2 + 2k - mn \in \mathbb{Z}$.

Case 2: $m + n$ is even

We can write $m + n = 2k$, for some $k \in \mathbb{Z}$. Similarly, we have,

$$\begin{aligned} m^2 + n^2 &= (m + n)^2 - 2mn \\ &= (2k)^2 - 2mn \\ &= 4k^2 - 2mn \\ &= 2(2k^2 - mn) \end{aligned}$$

which is even, as $2k^2 - mn \in \mathbb{Z}$.

We have shown that $m + n$ has the same parity as $m^2 + n^2$ and we are done.

Problem 2: Your favorite pizza place in the world, Elyssa's Pizzeria, is known for its variety of different pizzas. Elyssa's has 5 different kinds of tomato sauces and 6 different kinds of cheese. In addition, you can add one of any 20 different toppings, which are optional. On top of all of these choices, you can choose a thin crust or a thick crust. How many different pizzas can you possibly order from Elyssa's?

Solution:

We can apply the multiplication rule:

Step 1: Choose the sauce. (5 ways)
Step 2: Choose the cheese. (6 ways)
Step 3: Choose the topping. (20 toppings + 1 option for no toppings = 21 ways)
Step 4: Choose the crust. (2 ways)

Multiplying all of these, we get $5 \times 6 \times 21 \times 2 = \boxed{1260}$ pizzas.