CIS 160 Recitation 2

Counting, Sets, Proof Techniques

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Sets

- A *set* is an unordered collection of distinct objects. $\{a, b, c\}$
- Two sets are *equal* if and only if they have the same elements.
- The *cardinality* $|S|$ is the number of distinct elements in S.
- A is a *subset* of B ($A \subseteq B$) if and only if every element of A is an element of B.
- A *powerset* of S ($\mathcal{P}(S)$) is a set of all possible subsets of S.
- $\mathbb{N} = \{0, 1, 2, 3, ...\}, \mathbb{Z} = \{..., -2, -1, 0, 1, 2, ...\}, \mathbb{Q} = \{p/q \mid p \in \mathbb{Z} \wedge q \in \mathbb{Z} \wedge q \neq 0\}, \mathbb{R}$ is a set of real numbers.

Set Operations

- *Union*: $A \cup B$ contains elements either in A or B, or both.
- *Intersection*: $A \cap B$ contains elements in both A and B.
- *Disjoint*: A, B are disjoint if $A \cap B$ is an empty set.
- *Partition*: $\{A_1, A_2, ..., A_n\}$ is a partition of A if and only if (i) $A = \bigcup_{i=1}^{n} A_i$ and (ii) $A_1, A_2, ..., A_n$ are mutually (pairwise) disjoint.
- *Difference*: $A \setminus B$ or $A - B$ contains elements that are in A not but in B.
- *Complement*: $\bar{A}$ contains all elements not in A. $\bar{A} = U \setminus A$ when U is the universe of elements.
- *Cartesian Product*: $A \times B = \{(a, b) \mid a \in A, b \in B\}$

DeMorgan's Laws

- Let A, B, C be sets.

$$A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$$

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Counting

- **Multiplication Rule.** If a procedure can be broken down into k steps and the first step can be performed in n_1 ways, the second step can be performed in n_2 ways,..., the k^{th} step can be performed in n_k ways *regardless of how the preceding steps were performed*, then the entire procedure can be performed in $n_1 \cdot n_2 \cdot \ldots \cdot n_k$ ways.

- **Permutations.** A permutation of a set of distinct objects is an ordering of the objects in a row. By multiplication rule, the number of permutations of n distinct objects is $n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1 = n!$

Proof by Contradiction

- To prove that a claim p is true, assume towards contradiction that $\neg p$ is true. Then, we have to show that $\neg p$ is false by reaching a contradiction (some statement that is always false).
- **Example:** Prove that "if $3n + 2$ is odd then n is odd." We can show this by contradiction by assuming that "$3n + 2$ is odd and n is even" and showing that this statement is false. (Proof done in Lecture 2H)

Proof by Contrapositive

- To prove that $p \implies q$ is true, we can show that $\neg q \implies \neg p$ is true.
- **Example:** Prove that "if x and y are integers where $x + y$ is even, then x and y are both odd or both even". We can show this by proving the contrapositive, "if exactly one of x or y is even then $x + y$ is odd". (Proof done in Lecture 2H)