CIS 160 Recitation #1

Logic, Proofs, and Counting

Propositions

Proposition: A statement that is either true or false.

Examples of statements that are propositions:

- $2 + 2 = 4$
- "It is raining today in Philadelphia, PA."
- $2x = x + 5$ when $x = 4$

Examples of statements that are NOT propositions:

- $x^2 + 4 < 9$
  - This may or may not be true depending on the definition of x.
- $2x = x + 5$
  - This statement is not true or false. We are simply assigning a value of 5 to x.

Compound Propositions

Various connectives can form compound propositions from simpler propositions:

Negation: $\bar{p}$ (“not p”)

Conjunction: $p \wedge q$ (“p and q”)

Disjunction: $p \vee q$ (“p or q”)

Exclusive-or: $p \oplus q$ (“p exclusive-or q”)

Implication: $p \implies q$ (“p implies q”)

Biconditional: $p \iff q$ (“p if and only if q”)

Necessary and Sufficient Conditions

- "p is a sufficient condition for q":

$$p \implies q$$

- "p is a necessary condition for q":

$$q \implies p$$

- "p is a necessary and sufficient condition for q":

$$p \iff q$$

Logical Equivalence

- Two compound propositions are equivalent if they always have the same truth value.

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg p \vee q$	$\neg q \to \neg p$
T	T	F	F	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Quantified Statements

- Earlier, I told you that $x^2 + 4 < 9$ isn't a proposition. But, it becomes a proposition once we assign a value to x.
- Let our statement $x^2 + 4 < 9$ be P(x)
    - P(1) and P(2) are true, P(3) and P(4) are false. These are propositions.

Quantifiers:

$\forall$ Universal Quantifier ("for all")

$\exists$ Existential Quantifier ("there exists")

Examples of propositions using quantifiers:

$\forall x \in \mathbb{Z}, x^3 + 1$ is composite.

$\exists x \in \mathbb{Z}, x^2 \not> x$

Sets

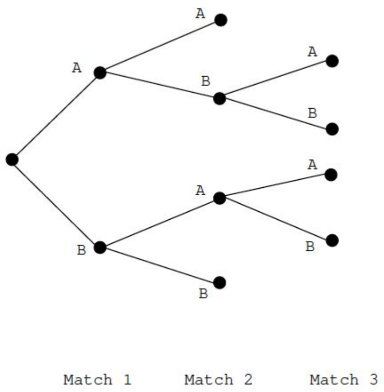
Set: **unordered** collection of **distinct** objects

- Two sets are equal if and only if they have the same elements
- The cardinality of a set S, denoted by $|S|$, is the number of distinct elements in S.
- A set A is a subset of a set B if all elements in A are in B ($A \subseteq B$).
- If $A \subseteq B$ and $A \neq B$, then A is a proper subset of B ($A \subset B$).
- The power set of a set S, denoted by $P(S)$, is the set of all subsets of S.
- Set builder notation

  Example: $\{x \in \mathbb{Z} | x < 100\}$ is the set of all integers less than 100.

Counting Techniques

Tree Diagrams example: Two teams, team A and team B, play a best-of-three match (first team to win two games wins).



6 outcomes

Multiplication Rule:

If a procedure can be broken down into k steps, and the first can be done n_1 ways, the second n_2 ways, …, and the kth n_k ways, and the number of ways to do each step is independent of the previous step, the procedure can be performed in

$$n_1 \cdot n_2 \cdot \ldots \cdot n_k \text{ ways.}$$

Permutations

A permutation of a set S is an ordering of the elements of S.

Example: $S = \{a, b, c, d, e\}$

To count the number of unique permutations of the elements of S, use the Multiplication Rule!

Step 1: choose a place for a (5 ways)

Step 2: choose a place for b (4 ways)

…

Step 5: choose a place for e (1 way)

By the MR, we have 5! permutations.