

# Recitation Guide - Week 12

---

**Topics Covered:** Total Expectation, Memoryless Property, Hall's Theorem

**Problem 1:**

For a geometric random variable  $X$  with parameter  $p$ , where  $n > 0$ , we have the memoryless property

$$\Pr[X = n + k \mid X > k] = \Pr[X = n]$$

The following is the definition of conditional expectation.

$$\mathbf{E}[Y \mid Z = z] = \sum_y y \cdot \Pr[Y = y \mid Z = z],$$

- a) Prove the law of total expectation below. Given any random variables  $X, Y$ , defined in the same sample space,

$$\mathbf{E}[X] = \sum_y \mathbf{E}[X \mid Y = y] \Pr[Y = y]$$

- b) Calculate the expectation of a geometric random variable with the memoryless property and the law of total expectation.

**Problem 2:**

Consider a normal chessboard (an  $8 \times 8$  grid). In each row and in each column there are exactly  $n$  pieces, where  $0 < n \leq 8$ . Prove that we can pick 8 pieces such that no two of them are in the same row or column.