CIS 160 Recitation #11

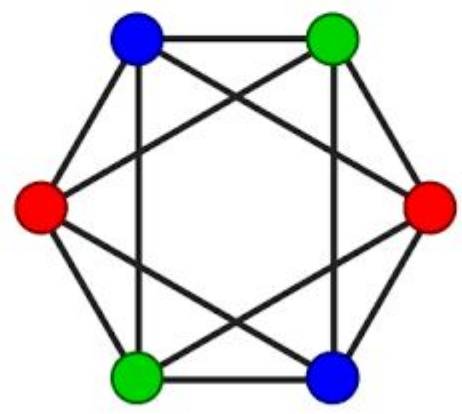
Graph Coloring, Matchings, Probability Distributions

Graph Coloring

A graph is **k-colorable** if each vertex can be colored using one of k colors such that adjacent vertices are colored using different colors.

Chromatic number: the smallest value of k for which a graph is k-colorable

Bipartite graph: a graph that is 2-colorable



Matchings

A **matching** M in a graph G is a set of edges with no shared endpoints.

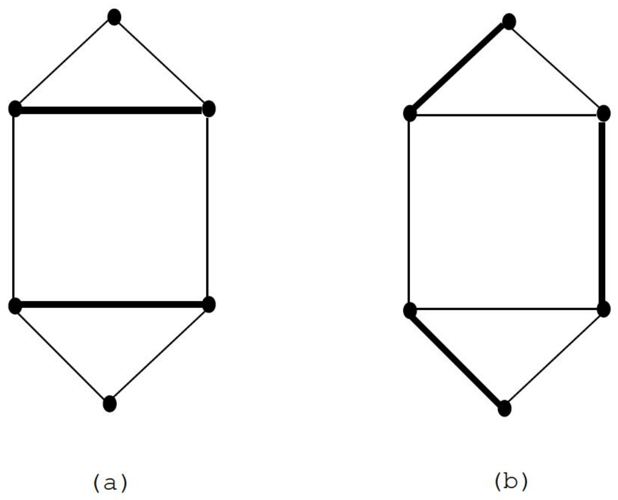
M-saturated vertex: a vertex that is incident on an edge in a matching M

M-unsaturated vertex: a vertex that is not M-saturated

A **maximal matching** in a graph is a matching that is not contained in larger matching. A **maximum matching** is a matching of maximum size among all matchings in the graph.

A **perfect matching** in a graph is a matching that statures every vertex.

Matchings Example



For each matching, is it maximal? Is it maximum? Is it perfect?

Variance

If X and Y are **independent**, real-valued random variables, then:

$$\mathrm{Var}[X + Y] = \mathrm{Var}[X] + \mathrm{Var}[Y] \text{ and } \mathbf{E}[X \cdot Y] = \mathbf{E}[X] \cdot \mathbf{E}[Y]$$

Probability Distributions

An experiment with exactly two outcomes, one of which has probability p, and the other probability 1 - p, is called a **Bernoulli trial**.

<u>Geometric Distribution</u>

- Repeat the experiment until first success
- What is the distribution of the number of times we run the experiment?

For a **geometric random variable** X:

$$\Pr[X = i] = (1 - p)^{i-1} p$$

$$\mathbf{E}[X] = \frac{1}{p}$$

Memoryless Property:

$$\Pr[X = n + k \mid X > k] = \Pr[X = n]$$

Probability Distributions (cont.)

Binomial Distribution

- Run the experiment n times
- What is the distribution of the number of successes?

For a **binomial random variable** X:

$$\Pr[X = j] = \binom{n}{j} p^j (1-p)^{n-j}$$

$$\mathrm{Var}[X] \quad = \quad np(1-p)$$

$$\mathbf{E}[X] \quad = \quad np$$