CIS 160

Recitation Guide - Week 11

Topics Covered: Relations, Probability Distributions, Probabilistic Method

Problem 1:

Consider a set A with $n \geq 1$ elements. We color independently each of the elements of A red with probability $\frac{1}{3}$ and blue with probability $\frac{2}{3}$. Let R be the "is the same color as" relation on A, ie. if a is the same color as b, then $(a, b) \in R$.

a) Is R an equivalence relation? If so, what are its equivalence classes?

b) Calculate the expected value of $|R|$.

Problem 2:
You are at an auction for a box of money. The amount of money in the box is unknown to you, and is secretly determined by Tien. Tien flips a biased coin 100 times, with a 1/3 chance of getting heads, and for each heads that appears, he puts a dollar in the box.

There is only one other bidder at the auction, Elyssa, who rolls a 6-sided fair die until she gets a 6, and for each roll adds \$5 to her bid.

Everyone who attends the auction reveals their bid at the same time, and the person with the highest bid pays that amount of money to get the box. (Assume you can only bet in whole dollar amounts.)

a) Let's say you want to bid strictly more than the expected value of Elyssa's bid (so you win the box), but strictly less than the expected value of the box (so you still make money). Is that possible?

b) What if Elyssa bids according to a 7-sided die, rolling until she gets a 7? In expectation and using the same strategy as a), can you still make money?

Problem 3:

Consider an undirected graph $G = (V, E)$, where $|V| = n$ and $|E| = m$. Prove that there is a partition of V into two disjoint sets A and B such that at least $\frac{m}{2}$ edges connect vertices across A and B.