

CIS 160

## Recitation Guide - Week 10

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**Topics Covered:** Eulerian Graphs, Linearity of Expectation, Graphs

**Problem 1:**

Given a directed graph  $G = (V, E)$ , prove that the sum of the outdegrees of all the nodes in a directed graph is equal to the number of edges.

**Problem 2:**

For any simple, undirected graph  $G = (V, E)$ , for any distinct  $u, v \in V$ , prove that if there is a walk from  $u$  to  $v$  then there is a path from  $u$  to  $v$ .

**Problem 3:**

Oliver, as a busy college student, hasn't done laundry in weeks. In particular, he realizes that he has no more socks to wear, so he goes to the laundry room and throws in his 2 distinct pairs of socks. However, the machine is broken, and so it only returns 2 of his socks at random! Note that the two socks in a pair are also distinguishable.

- (a) What is the expected number of pairs that he can wear now?
- (b) What if he throws in  $n$  pairs and only gets  $k$  ( $k > 1$ ) socks back?

**Problem 4:**

Prove that if a graph has an Eulerian circuit, its edges can be partitioned into a set of edge-disjoint cycles (that is, cycles that do not share any edges).