CIS 160 Recitation 10

Chebyshev's Inequality, Binomial and Geometric Distribution, Hall's Theorem

November 11-12, 2021

Chebyshev's Inequality

Let X be a random variable. For all $a > 0$:

$$Pr\Big[|X - E[X]| \geq a\Big] \leq \frac{Var[X]}{a^2}$$

Geometric Distribution

- A sequence of independent trials **until the first success** where each trial succeeds with a probability p.
- Suppose we have a biased coin with heads probability p that we flip until it lands on heads.
- $\Omega = \{H, TH, TTH, TTTH, ...\}$
- For any $\omega \in \Omega$ of length i, $Pr[\omega] = (1-p)^{i-1}p$.

Geometric Distribution

- A geometric r.v. X with parameter p has the following distribution for $i = 1, 2, \ldots$

$$Pr[X = i] = (1 - p)^{i-1} p$$

- $E[X] = \frac{1}{p}$ and $Var[X] = \frac{1-p}{p^2}$
- **Memoryless Property.** For geometric r.v. X with parameter p and for $n > 0$ and $k \geq 0$,

$$Pr[X = n + k \mid X > k] = Pr[X = n]$$

Binomial Distribution

- A sequence of n coin flips in which the probability of obtaining heads is p. How many flips result in head?

- A binomial r.v. X with parameter n and p has the following distribution for $j = 0, 1, 2, ..., n$:

$$Pr[X = j] = \binom{n}{j} p^j (1 - p)^{n-j}$$

- $E[X] = np$ and $Var[X] = np(1 - p)$

Hall's Theorem

- Let $G = (X, Y, E)$ be a bipartite graph. For any set S of vertices, let $N_G(S)$ be the set of vertices adjacent to vertices in S.
- G contains a matching that saturates every vertex in X iff $|N_G(S)| \geq |S|, \forall S \subseteq X$. (Hall's condition)