

Homework 9T

Due: October 29, 2020

This assignment is due at the beginning of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in []. To receive full credit, all your answers should be carefully justified; in particular, please make sure to explicitly define your sample space for any probability question unless otherwise specified. Each solution must be written independently by yourself - see Piazza for the course collaboration policy.

Also, please remember to double check that you have submitted the correct version of your homework onto Gradescope by re-downloading it.

1. [10 pts] FJared and FJarett's Forbearing Fructose Fetching Frenzy

Jared and Jarett are spending their Halloween trick-or-treating around their neighborhood. Their neighborhood has n houses, some of which are connected by two-way streets. Each street is between exactly two houses, and no two houses are connected by more than one street. Jared and Jarett want to plan their trick-or-treating in such a way that they park their car and start at an arbitrary house, get candy from at least one other house, then return back to the original house to get back in their car. As is proper trick-or-treater etiquette, they will not re-visit any house or road. Suppose that on every trick-or-treating route, Jarett and Jared will always get candy from an odd number of houses. Prove that there is a house with at most 2 streets that connect it to other houses.

2. [8 pts] Freaky Four-Suited Face-Down Flip-Over

Anni, Daniel, and Jasmine are playing a card game with a special Halloween-themed card deck. Instead of the traditional suits, the four suits in this card deck are Ghosts, Mummies, Vampires, and Zombies. Anni has 3 Ghosts, 1 Mummy, and 4 Vampires. Daniel has 7 Ghosts, 1 Vampire, 1 Mummy, and 2 Zombies. Jasmine has 3 Ghosts, 2 Vampires, and 2 Zombies. Each player independently makes their move by choosing one card from their respective hand uniformly at random and placing it face-down on the table. After everyone does this, all three cards are flipped face-up. What is the probability that Anni played a Ghost, given that there are exactly 2 Ghosts face-up on the table?

3. [12 pts] Fabled "Fantom" Formation From Fortuitous Fair Flips

A CIS 160 Haunted Halloween game consists of all TAs flipping 8 fair coin in succession. In the event that all 8 coins on a TA's turn land on heads, legend has it that that TA will turn into a ghost on October 31. Let us define the following events in the resulting probabilistic space:

A : event that the first toss result in Tails.

B : event that the fourth toss results in Heads.

C : event that there are a total of 4 Heads.

D : event that the longest repeated sequence (sequence of repeated Heads or Tails) is at least length 2.

E : event that the longest repeated sequence (sequence of repeated Heads or Tails) is at most length 7.

Answer the following questions giving proper justification.

- (a) Are events A and B independent?
- (b) Are events A and C independent?
- (c) Are events B and C independent?
- (d) Are events A , B , and C mutually independent?
- (e) Are events D and E independent?