CIS 160 Assigned: October 22, 2020

Homework 8H

Due: 9:00am EDT, October 27, 2020

This assignment is due at the beginning of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in [ ]. To receive full credit, all your answers should be carefully justified; in particular, please make sure to explicitly define your sample space for any probability question unless otherwise specified. Each solution must be written independently by yourself - see Piazza for the course collaboration policy.

Also, please remember to double check that you have submitted the correct version of your homework onto Gradescope by re-downloading it.

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1. [12 pts] Colin and the Every Flavor Beanstalk

Colin loves jelly beans and has bought a truckload of different jelly beans to play with. Please answer the following questions **(no work is required)**.

For Questions (a)-(d), Colin has four buckets labeled by the flavors of the jelly beans inside (Toasted Marshmallow, Berry Blue, Peach, and Juicy Pear). However, some of the other TAs have played a prank on him. Some of the jelly beans in the Toasted Marshmallow bucket are actually Stink Bug flavored, some of the jelly beans in the Berry Blue bucket are actually Toothpaste flavored, some of the jelly beans in the Peach bucket are actually Barf flavored, and some of the jelly beans in the Juicy Pear bucket are actually Booger flavored! Note that within each bucket, the jelly beans are indistinguishable, so Colin cannot tell apart the jelly beans with the labeled flavors and the jelly beans that the other TAs have put in. Suppose that only 84% of the jelly beans in the Toasted Marshmallow bucket are actually Toasted Marshmallow, only 62% of the jelly beans in the Berry Blue bucket are actually Berry Blue, only 57% of the jelly beans in the Peach bucket are actually Peach, and 0% of the jelly beans in the Juicy Pear bucket are actually Juicy Pear (the TAs were extra evil). Colin selects exactly 1 bean from each of the buckets independently and uniformly at random.

Questions (e)-(i) are based on the following scenario: among his jelly beans, Colin discovers 5 magic beans. He plants each of them in his backyard, and each of them is equally likely to grow either a tall beanstalk or a small, wilting plant.

(a) What is the probability that exactly 3 of the jelly beans are the correct labeled flavor?

(b) What is the probability that exactly 0 of the jelly beans are the correct labeled flavor?

(c) What is the probability that at least 1 of the jelly beans is the correct labeled flavor?

(d) What is the probability that exactly 1 of the jelly beans is the correct labeled flavor?

(e) What is the probability that all 5 of the magic beans grow tall beanstalks?

(f) What is the probability that exactly 3 of the magic beans grow tall beanstalks?

(g) What is the probability that at most 2 of the magic beans grow tall beanstalks?

(h) Given that at least 2 of the magic beans grow tall beanstalks, what is the probability that at least 4 of the magic beans grow tall beanstalks?

(i) Given that at least 1 of the magic beans will grow a small, wilting plant, what is the probability that at least 3 of the magic beans grow tall beanstalks?

2. [8 pts] Trick Tac Toe

Consider a large haunted house represented by a 3×3 grid composed of nine unit square rooms. Your friend tells you there is a 50 percent chance that in any room you'll find Pennywise ("trick") and 50 percent chance you'll find a a lifetime supply of jolly ranchers ("treat"). Find the probability that the houses have at least one 2×2 collection of rooms that contain "treats"?

3. [10 pts] <insert (candy) corny title here>

After a long night of socially distanced trick-or-treating, Niko, Olivia, Jasmine, and Andy find that they've each collected 160 pieces of giant candy corn. Each candy corn has a color and a number label associated with it. To keep track of them, each person labels their candy corn from 1 to 160, but Niko and Olivia have orange candy corn, and Jasmine and Andy have white candy corn.

So that they can tell their candy corn apart, Niko and Jasmine put all of their candy corn into one pillowcase, and Olivia and Andy put all of their candy corn into another one. Now, we pick one candy corn uniformly at random from each pillowcase.

(a) Given that at least one of the two candy corn selected is orange, what is the probability that both candy corn chosen are orange?

(b) Given that at least one of the two candy corn selected is an orange candy corn labeled with 37, what is the probability that both candy corn are orange?

4. [10 pts] Super Scary Head TA Kara's Sometimes-Successful Saturday Spooking Spree

Every Halloween, Super Scary Head TA Kara unleashes her scary side by trying to spook the other TAs. However this semester, with everything being online, she finds that spooking people through Zoom is a more difficult task since the other TAs can just mute Kara and ignore her. Because of this, the probability that she successfully spooks any given TA is only $\frac{1}{2}$. Given that she attempts to spook 6 TAs on Friday and 5 TAs on Saturday, what is the probability that Super Scary Head TA Kara will end up successfully spooking more TAs on Friday than

Saturday?

5. [6 pts] Lame Labor for the Last in the Labyrinth

The TAs have just finished building their annual Halloween Labyrinth. It consists of $r+1$ rooms connected by r tunnels ($r \in \mathbb{N}$), and any pair of rooms can have at most one tunnel between them. The Labyrinth is patrolled by Kreepy Kara, scariest of all the TAs. If she catches you before you escape, she makes you prove the following:

There must be at least one room in the Labyrinth that is connected to at most 1 other room. Prove it before it's too late!

6. [12 pts] Sid's Series of Super Special Spooky Sidewalks

Sid loves haunted houses! There are a total of $n \geq 3$ haunted houses in Sid's neighborhood, but Sid can only travel between haunted houses through haunted underground tunnels. Every tunnel connects exactly 2 haunted houses. Also, Sid is able to travel from any haunted house to any other haunted house via a series of tunnels. Sid wants to embark on a loop of haunted houses, such that he visits some number of haunted houses before returning to the one he started from and he visits each haunted house on the loop only once.

Prove that there exists exactly one such loop if and only if there are exactly n total haunted tunnels in the neighborhood.

7. [12 pts] Terrible Task for Taki while Trapped in Tortuous Tricky Twists

While patrolling the newly built Halloween Labyrinth, Kreepy Kara captures Terrified Taki before he could escape! Upon realizing Taki was missing from the group, the other TAs muster up the courage to brave the labyrinth to find him. However, Kreepy Kara set up an additional challenging problem before the entrance as a test of staff loyalty and bravery. Help the TAs answer the problem to protect Taki at all costs!

Let T be a tree with $n > 1$ vertices. Prove that the number of leaves in T must be:

$$2 + \sum_{\substack{v_i \in V, \\ \deg(v_i) \geq 3}} (\deg(v_i) - 2)$$