

# Homework 6t

Due: 9:00 am EDT, October 8, 2020

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This assignment is due at the beginning of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in [ ]. To receive full credit all your answers should be carefully justified. Each solution must be written independently by yourself - no collaboration is allowed.

Also, please remember to double check that you have submitted the correct version of your homework onto Gradescope by re-downloading it.

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1. [15 pts] Sid, looking to fulfill his duties as head TA, decided to attend the Socially Distanced Stack Exchange Educator's Conference, which had  $n \geq 2$  people in attendance. Before discussing how to prevent students from posting homework questions on Stack Exchange, the attendees began greeting other attendees with cordial elbow bumps. Note that any attendee can elbow bump any number of other attendees (including zero), but any two attendees can only elbow bump once. Moreover, each elbow bump is a mutual event between exactly two attendees.

In an attempt to stay awake, Sid carefully noted every elbow bump that took place and noticed that two attendees gave the same number of elbow bumps. Prove or disprove the claim that there must always be at least two attendees who have elbow bumped the same number of attendees.

2. [15 pts] The Fibonacci numbers are defined as follows:

$$F_1 = 0$$

$$F_2 = 1$$

$$F_n = F_{n-2} + F_{n-1}, \forall n \geq 3$$

Prove using induction that any natural number can be expressed as a sum of one or more distinct Fibonacci numbers, no two of which have consecutive Fibonacci indices. For example,  $71 = F_{11} + F_8 + F_5 = 55 + 13 + 3$ .