

Homework 6H

Due: 9:00am EDT, October 13, 2020

This assignment is due at the beginning of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in []. To receive full credit all your answers should be carefully justified. Each solution must be written independently by yourself - no collaboration is allowed.

Also, please remember to double check that you have submitted the correct version of your homework onto Gradescope by re-downloading it.

1. [23 pts] That's Some Bull!

Taking influence from the Texan TAs, Kadin visits the Texas rodeo. At the rodeo, Kadin chooses to ride one of the $n \geq 4$ bulls. However, at this unconventional rodeo, some pair of bulls x, y are connected by lassos which has one end on bull x and one end on bull y . Kadin notices that among any group of 4 bulls, there will always be at least one bull in the group that has a lasso from it to each of the other bulls. Help Kadin prove or disprove that among n bulls with the above property, there exists a bull b such that for every other bull b' , b and b' have a lasso between them.

2. [23 pts] short rib shebang... BBQ BANGER!!!

Olivia Z. is renowned throughout the state of Texas for her Korean barbecuing prowess. Every Friday after teaching recitation, she fires up the grill to make her famous marinated beef short rib for lunch. On the first week, she is a bit out of practice due to quarantine, and only makes $n_1 = 2$ pounds of short rib. In her second week, she makes $n_2 = 1 \times n_1$ pounds of short rib. After two weeks, Olivia Z. finds her groove, and in the third week, she makes $n_3 = 2 \times n_2 + 1 \times n_1$ pounds of short rib. In the fourth week, she makes $n_4 = 3 \times n_3 + 2 \times n_2 + 1 \times n_1$ pounds of short rib. Her barbecuing habits continue as such in all the following weeks, that is, in the z^{th} week (for $z \in \mathbb{Z}, z \geq 2$), she makes $n_z = (z - 1) \times n_{z-1} + (z - 2) \times n_{z-2} + \dots + 1 \times n_1$ pounds of her famous marinated beef short rib.

Give a simple closed-form expression for $n_z (z \in \mathbb{Z}^+)$ and prove it using induction.

3. [24 pts] Sneaky Stack Exchange Slide In

Dried out from the Texas heat, Matt wants to talk to his crush in the Stack Exchange DM's (which he definitely doesn't use for CIS 160 answers). However, he remembers that he still has to do his homework. Help Matt prove the following inequality for all $n \in \mathbb{Z}^+$:

$$\prod_{i=1}^n \frac{2i-1}{2i} \leq \frac{1}{\sqrt{3n}}$$

- (a) Suppose that we want to prove this statement using induction. Can we let our induction hypothesis simply be the above assertion? Show why this does not work by trying to prove it using induction.
- (b) Try proving the claim by instead strengthening the induction hypothesis by changing $3n$ to $3n + 1$ in the above assertion. In other words, prove:

$$\prod_{i=1}^n \frac{2i-1}{2i} \leq \frac{1}{\sqrt{3n+1}}$$

- (c) Does proving the new claim in (b) imply what you were trying to prove in part (a)?