

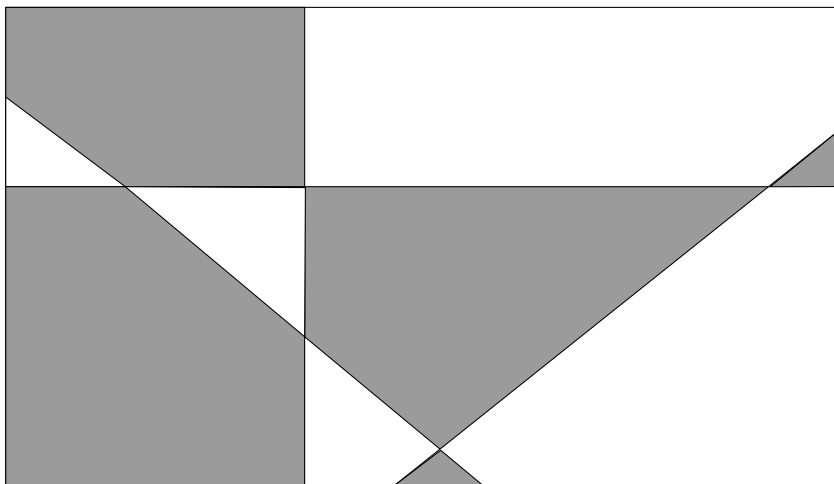
Homework 5H

Due: 9:00am EDT, October 6, 2020

This assignment is due at the beginning of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in []. To receive full credit all your answers should be carefully justified. Each solution must be written independently by yourself - no collaboration is allowed.

Also, please remember to double check that you have submitted the correct version of your homework onto Gradescope by re-downloading it.

- [10 pts]** In between practices in the bubble, LeBethany James, Andythony Davis, and Alex Karaso decide to freshen up their 160 skills and work together to define an arbitrary set S such that $\forall x \in S, x \in \mathbb{Z}^+$ and $|S| = 160$. Kadinvious Caldwell-Pope finds out that if there are positive integers $a, b \in S$, (where $a > b$), such that $100 \mid (n^a - n^b)$ for any positive integer n , then the Lakers will win the championship. Prove that the Lakers will win the championship!
- [12 pts]** Zach is upset because he's just too garbage at basketball. As such, he decides to invent a new sport called 160-ball that (he thinks) all the cool kids will play. He decides to create the court for the game by taking a rectangle and drawing $n \geq 0$ straight lines such that no two lines share the same two endpoints. Each line also begins on one edge of the court and ends on another. These lines divide the court into polygonal regions that don't necessarily have to be rectangles. Zach, being the nerd that he is, observes that the regions of the court he formed by drawing the n lines can be colored using two colors in such a way that two regions sharing a boundary line are colored using different colors. Prove using induction that Zach's observation holds no matter how many lines he draws (regions that touch each other at only their corners are allowed to have the same color).



An example of one such court with $n = 4$ lines

3. [12 pts] Taki has been declared manager of the filming/technical crews of the NBA. He believes there has actually been an increase in the funding for technical staff across the NBA during the past year! Woohoo! Last year, for each of the $n \geq 1$ teams in the NBA, team i had allocated $\frac{1}{\sqrt{i}}$ dollars for their technical staff. However, this year, the NBA technical staff funding totals $2\sqrt{n}$ dollars. Help Taki by proving using induction that there was in fact an increase in total funding of technical staff between last year and this year. That is, show that

$$\sum_{i=1}^n \frac{1}{\sqrt{i}} < 2\sqrt{n}$$

4. [12 pts] A big basketball event is happening soon! Obviously, we are talking about the CIS 160 TA Basketball League finals, DUH! It's the biggest basketball event of the year! The event consists of $n \in \mathbb{Z}^+$ scrimmages, and every fan attending the event will attend at least one scrimmage, socially distanced of course! Prove by induction that the total number of fans attending is at most the sum of the number of fans attending each scrimmage.

Please make sure to first phrase the claim using mathematical notation, and only proofs (using mathematical notation) by induction will receive credit.

5. [12 pts] After the Celtics lost in the Eastern Conference Finals, they decide to take their time off to travel the world. Lucky for them, Billy is the CEO of Billy's B-rated Budget Airline and says he will allow the Boston Celtics to fly for free.

Billy's B-rated Budget Airline services $n \geq 2$ airports. Of those airports, there exists a scheduled flight route between any two airports S, T . The flight can either go only from S to T , only T to S , or in both directions.

Being a new (and pretty bad) airline, Billy's B-rated Budget Airline has a problem: some airports might be dead-ends, meaning that there are no scheduled flights departing from the airport!

Before the Boston Celtics decide whether or not to take Billy's offer, they are allowed to ask at most $2(n-1)$ questions to Billy. The questions have to take the form of "Can we fly from airport S to airport T ?" Billy will truthfully answer either "Yes" or "No". Prove, using induction, that if there are n airports, the Boston Celtics can find a dead-end airport, if it exists.

6. [12 pts] Atharva is super sad because his favorite team, the Celtics, didn't make the NBA Finals, and he determines that the only activity to keep him preoccupied is to think about placing all of the basketball players on a coordinate plane. He decides that on each point (a, b) where $a, b \in \mathbb{Z}$ on the (x, y) -coordinate plane, there will be exactly one player that is wearing

1 of 3 jerseys - a green jersey, a black jersey, or a white jersey. While placing the basketball players on the plane, he notices that regardless of their arrangements, he can always create a rectangle in which the players at each corner are wearing a jersey of the same color. Prove that Atharva's observation is true!

(Hint: Consider the number of different ways players can stand on 4 consecutive points.)