

Homework 4T

Due: 9:00 am EDT, September 24, 2020

This assignment is due at the beginning of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in []. To receive full credit all your answers should be carefully justified. Each solution must be written independently by yourself - no collaboration is allowed.

1. [18 pts] Answer each of the following questions. You do not need to show your work.
 - (a) **True or False?** Adi has decided to run for city council, and he prints up 30 premium flyers to distribute. His goal is to distribute these indistinguishable flyers to 44 houses on his block. Assuming that the houses are distinguishable, and each house can receive more than one flyer, there are $\binom{73}{30}$ different ways that the flyers can be distributed.
 - (b) **True or False?** In the above question, if the flyers and houses both are distinguishable, then the answer is 44^{30} .
 - (c) **True or False?** In the above question, if neither the flyers nor the houses are distinguishable, then there is only 1 way in which the flyers can be distributed.
 - (d) Suppose Niko received 13 buttons from Adi's campaign and he wants to store them in his room. He has a large shelf that consists of many compartments that form a 13×13 grid. Because Niko appreciates a good problem, he wants to make sure that no two buttons are in the same row and that no two buttons are in the same column. How many ways can he place the buttons? Assume that the buttons are indistinguishable.
 - (e) Answer (d) if the buttons are all distinguishable.
 - (f) Answer (d) if there are 2 red buttons, 4 blue buttons, 6 green buttons, and 1 purple button. Assume that buttons of the same color are indistinguishable, but buttons of different colors are distinguishable.
 - (g) How many different outcomes are possible if 7 identical dice are rolled?
 - (h) How many different outcomes are possible if 14 identical coins are flipped?
 - (i) Oliver has strawberry, pumpkin, rum-raisin, and grape flavored protein powders, and 8 scoops of each type of protein powder. Oliver will eat exactly one scoop of protein powder each day over the next 32 days, and must not eat the same flavored protein powder on the first and last day, otherwise his gains would be ruined. Given that protein powders of the

same type are indistinguishable, how many ways can Oliver eat the protein powders and get swole?

- (j) How many permutations of length 8 are there of the 9 letters of the word **addresses**
- (k) Determine the number of ways to distribute 17 bowling balls, 1 Boston cream pie, 1 Blu-ray player and 1 Bildungsroman hardcover to 5 hardworking students so that each student gets at least 1 item, and the Boston cream pie, Bildungsroman, and Blu-ray player each go to different students.
- (l) Find the number of solutions to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 721$$

in which each of $x_1, x_2, x_3, x_4, x_5, x_6$ and x_7 is a positive odd integer.

2. [12 pts] Using induction, please prove the following:

- (a) For all $n \in \mathbb{Z}^+$,

$$\sum_{i=1}^n i \cdot i! = (n+1)! - 1$$

- (b) For all $n \in \mathbb{Z}^+$,

$$\sum_{i=1}^n i(i+1)(i+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$