

Homework 2T

Due: 9:00 am EDT, September 10, 2020

This assignment is due at the beginning of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in []. To receive full credit all your answers should be carefully justified. Each solution must be written independently by yourself - no collaboration is allowed.

1. [8 pts] Prove the following.

- Let $X = \{m \in \mathbb{N} \mid m = 6t - 5, \text{ for some } t \in \mathbb{N}\}$ and $Y = \{m \in \mathbb{N} \mid m = 24t - 5, \text{ for some } t \in \mathbb{N}\}$. Prove that $Y \subset X$.
- Let $X = \{m \in \mathbb{N} \mid m = 10t - 5 \text{ and } t \in \mathbb{Z}^+\}$ and $Y = \{m \in \mathbb{N} \mid m = 10t + 5, \text{ for some } t \in \mathbb{N}\}$. Prove that $X = Y$.
- Let $X = \{m \in \mathbb{N} \mid m = 9t + 4, \text{ for some } t \in \mathbb{N}\}$ and $Y = \{m \in \mathbb{N} \mid m = (2t - 1)^3, \text{ for some } t \in \mathbb{N} \text{ and } t \geq 3\}$. Prove that $X \neq Y$.

2. [12 pts] Prove or disprove the following.

- For every prime p , either $p + 10$ is a prime or $p + 11$ is a prime.
- $\forall x, y \in \mathbb{R}^+, \lfloor x + y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$.
- $\forall x \in \mathbb{N}, (x + 1)^3 - 5x^2 + 6$ is odd.
- $\forall q, r, x, y, s \in \mathbb{Z}, (s|q \wedge s|r) \implies s|(qx + ry)$.
- $\forall i, j, k \in \mathbb{Z}$, if $i - j$ is even and $j - k$ is odd, then $i - k$ is odd.
- Let w be a positive integer and r be a non-negative, real number. Prove that

if r is irrational, then $r^{1/w}$ is irrational.

3. [10 pts] Kadin is looking to get revenge on the other CIS 160 TAs for the humiliating defeat that he suffered in last spring's CIS 160 water balloon fight. He assembles an arsenal of 59 distinguishable water balloons. Within these 59 water balloons, there are 12 disjoint (non-overlapping) pairs of balloons such that

- one balloon in the pair is green, and the other balloon in the pair is orange, and
- the pair of balloons will only be effective if each balloon in the pair is placed directly next to the other. That is, if the two balloons are not adjacent, then they will lose their effectiveness.

In preparation for the upcoming water balloon fight, Kadin looks to arrange his balloons in a line such that all balloons will be effective.

- (a) Jarett, Kadin's teammate, suggests that in order to maximize effectiveness, the green balloon should be placed directly to the left of the orange balloon in each of the aforementioned 12 pairs. Given this, how many ways can Kadin arrange the 59 balloons?
- (b) Kadin ends up rejecting Jarett's advice, figuring that the order of the green and orange balloon in each of the 12 disjoint pairs of balloons doesn't matter; as long as each green balloon in a pair is placed either to the right or left of its orange counterpart, both balloons in the pair will be effective weapons. How many ways are there now to order all of the balloons?