CIS 160 Assigned: September 10, 2020

Homework 2H

Due: 9:00 am EDT, September 15, 2020

This assignment is due at the beginning of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in [ ]. To receive full credit all your answers should be carefully justified. Each solution must be written independently by yourself - no collaboration is allowed.

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1. **[10 pts]** "Man, office hours really make me thirsty," Oliver exclaimed as he reached for his Poland Spring. A strange section on the label made him pause...
"Suppose that $d, t \in (\mathbb{R} \setminus \{0\})$. Prove that

$$d < \frac{1}{d} < t < \frac{1}{t} \implies d < -1$$

for a chance to win a lifetime supply of Poland Spring." Help Oliver win his lifetime supply so he can stay hydrated during office hours.

2. **[16 pts]** Will has dug a bunch of deep wells around Penn's campus to power his hydroelectric underground bitcoin mine. Let S be the set of Will's soggy wells, T be the set of Will's thick wells, and R be the set of Will's ramshackle wells. Prove or disprove the following (note, a well could be in more than one of these sets).

(a) $S \setminus (T \setminus R) = (S \setminus T) \setminus R$

(b) $S \setminus (T \setminus R) \subseteq (S \setminus T) \cup R$.

(c) $S \setminus R = (S \setminus (T \cup R)) \cup ((S \cap T) \setminus R)$

(d) Prove that if $\exists$ an ordered pair of wells y such that $y \in S \times T \land y \in T \times S$, $\exists$ a well x such that $x \in S \land x \in T$.

3. **[8 pts]** Over quarantine, Ziya discovered her new passion - collecting jiggly water beds. By pure coincidence, Kara happens to be selling a bulk deal of 8 (distinguishable) jiggly water beds in different colors. Ziya decides to purchase all 8 of Kara's jiggly water beds for her 3 (distinguishable) bedroom apartment. Ziya wants to put at least one jiggly water bed in each of her bedrooms. How many ways can Ziya place her jiggly water beds in her bedrooms? Note that all jiggly water beds must be placed in a bedroom. Make sure you use the Principle of Inclusion-Exclusion (PIE) in your answer.

4. **[8 pts]** Some of the TAs have decided to live together in Arizona this semester. When they get to the Airbnb, they find out that the house has 289 swimming pools in the backyard arranged in a 17×17 grid.

(a) Upon seeing this nice surprise, Linda has an idea to buy 17 indistinguishable unicorn floaties to place into distinct pools in the grid. How many ways can she arrange the 17 unicorn floaties so that no two are in the same row or column?

(b) Linda tells the others her plan, but Kadin thinks it would be more fun to buy 17 different floaties instead. How many ways can they arrange distinguishable floaties in the pools such that no two are in the same row or column?

5. [12 pts] David has decided to stop being a 160 TA and instead follow his innate entrepreneurial desires. Thinking that regular old water is too boring and tasteless, he developed the innovative idea to sell colored water instead! (Ingredients: water, food coloring). He decides he will sell 4 different types of water: red, blue, green, and yellow. To spice things up even more, he decides to use colorful bottle caps as well. The bottle caps themselves are also red, blue, green, and yellow. However, David hates things that match, so he refuses to sell any bottles of water where the cap is the same color as the water it contains. For example, David will not sell a bottle with a yellow cap that contains yellow water, but will sell a bottle with a red cap that contains yellow water.

(a) David wants to sell bottles in packs of 4 such that each pack contains a bottle of water of all 4 colors, as well as a bottle cap of all 4 colors. How many possible packs of water can he create? Assume that the order of the bottles doesn't matter.

(b) After finding huge initial success, David decides to expand his business. He is now able to make water in 10 different colors, as well as bottle caps in those same 10 colors. He also decides to sell his bottles in packs of 10 now. How many different packs of water can David make now? He still wants one bottle cap of each color, as well as water of each color in his pack.

6. [8 pts] Arnold and his friends are planning on going to a water-park adventure with a total of n people. When going on the Super Soaker Group Slide, Arnold realizes that there are actually many slides going down. Since some of his friends are flaky he only knows that at least 18 people are present. Note that each person can only go down at most one slide, and some people may not get to ride any slides.

(a) In how many ways can groups be organized if there are a total of 3 indistinguishable slides and exactly 6 people are allowed on each slide.

(b) Oh no! Each slide now has a different maximum person limit, and each slide has to be used at its maximum capacity. The Violet KABOOM Slide has a limit of 6, the Green EXPLOSION Slide has a limit of 5, and the Orange SPLASH Slide has a limit of 4. How many different arrangements are possible?

7. **[8 pts]** The CIS 160 staff is enjoying a day at the beach. Bethany, Saurabh, Will, and Ziya are playing a competitive game of Spikeball. Wanting the rest of the staff to join in on the fun, they decided to assemble k identical Spikeball nets so everyone can play. They notice that for each net, the pieces of string making up the circular net originate at n points on the circle. Between any two points on each net, there is a tightly bound piece of string (note: this means that $n-1$ pieces of string originate at each point on each net). Across all k nets, in how many total points do two pieces of string intersect? Each string on a given net is at the same height. You should assume that there is no point on any net where three pieces of string intersect.