Homework 1H

This assignment is due at the beginning of the the first section on the due date. Unless all problems carry equal weight, the point value of each problem is shown in [ ]. To receive full credit all your answers should be carefully justified. Each solution must be written independently by yourself - **no collaboration is allowed**.

1. **[12 pts]** Give answers to the following questions. For this question only, no justification is required as a part of your answer.

   (a) List the members of these sets.

      i. $\{x \mid x$ is a positive integer that is not prime and less than $20\}$

      ii. $\{x \mid x \in \mathbb{N}, x \leq 20 \wedge (3|x \oplus 4|x)\}$

      iii. $\{x \mid \exists y \in \mathbb{Z}^+ \wedge \exists z \in \mathbb{Z}^+ \text{ s.t. } x = y(y+z) \wedge x < 15\}$

      iv. $\{x \mid x \subseteq \{a, b, c, d\} \wedge \{a, b\} \subseteq x\}$

   (b) Use the set builder notation to give a description of each of these sets.

      i. $\{2, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \frac{6}{25}\}$

      ii. $\{15, 18, 24, 27\}$

      iii. $\{0, 3, 8, 15, 24, 35\}$

   (c) What is the cardinality of each of the following sets?

      i. $\emptyset$

      ii. $\{\{\{x\}, y, \{a, b, c\}\}\}$

      iii. $\{a, \{a\}\}$

      iv. $\{\emptyset\}$

      v. $\{\{x, \{y\}\}, \{a, \{a\}\}, \{a, b, c\}\}$

   (d) Determine whether each of the following is true or false.

      i. $\emptyset \in \emptyset$

      ii. $\emptyset \subseteq \emptyset$

      iii. $\{1, 2, 3\} \subseteq \{1, 2, \{3\}\}$

    iv. $\emptyset \subseteq \{\{\emptyset\}\}$

    v. $\emptyset \in \mathcal{P}(\emptyset)$

    vi. $\emptyset \subseteq \mathcal{P}(\emptyset)$

(e) Let A be the power set of $\{1, 2, 3\}$. What are A and $|A|$?

(f) Find two sets A and B such that $A \in B$ and $A \subseteq B$.

2. **[10 pts]** Atharva decides to write a poem for his secret lover. Appreciative of the free form and rigid structure of poetry, he decides that every word in his poem must be exactly 13 characters, which can be uppercase letters or lowercase letters in the English alphabet (a-z, A-Z). (Note: in this context, a 'word' is simply any sequence of 13 characters, not necessarily in the English dictionary).

(a) Given this, how many unique words are possible for Atharva's poem?

(b) How many of these words are composed solely of the 5 vowels?

(c) How many of these words are composed solely of the 21 consonants?

(d) Atharva got curved after his first poem and decides to change up his style. He decides that each word must contain at least one vowel and at least one consonant. Given this, how many unique words are now possible in Atharva's poem?

For this question, consider "y" to be a consonant.

3. **[10 pts]** Now that Atharva has successfully wooed his lover, he wants to create a Twitter username for them. All usernames begin with '@', and each username may consist of the 26 letters in the English alphabet (in uppercase or lowercase), the 10 digits, and a single hyphen (-). The hyphen may or may not be used, but if it is used, it must be between other characters, rather than beginning or ending the username. Furthermore, a username must consist of 13 distinct characters, excluding the '@'. Uppercase and lowercase letters are considered distinct (e.g. 'a' is distinct from 'A'). For example, @160-PigeonsYO is a valid username, @-160ROCKSSSSS is not. How many distinct usernames can Atharva create?

4. **[10 pts]** Linda has moved into a new apartment for the fall semester and is trying to decorate her living room. She decides to paint her wall in the design of 6 vertical stripes placed above 3 horizontal stripes, as shown in the picture below. She has five different colors of paint: white, black, red, green, and gold. She can use the same paint color multiple times, but she doesn't want two stripes of the same paint color to share a long edge.

Being a CIS 160 TA, Linda wonders how many design combinations she could make. Can you help her count the number of possible designs that fit her description?
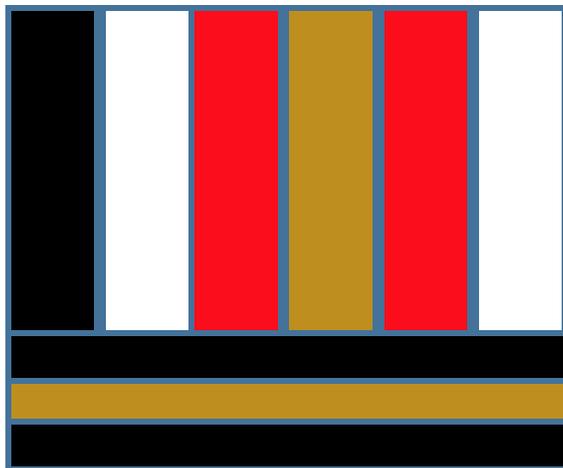
As an example, here is one such possible wall design:

5. **[8 pts]** Taki and Serena arrived back on campus to start the new school year and want to catch up at dinner. They are trying to decide where to eat and to help them decide, Taki writes the numbers 1 through 2020 on the whiteboard. They then take turns doing the following: on each turn, they choose two numbers to erase and replace the two numbers with their positive difference (they add back the positive difference just once). They repeat this until a single number remains. If that number is a 1, they will go to 1920 Commons; otherwise, they will eat at Hill College House. Where will Taki and Serena eat? Explain your answer. (Note: You should not be using a calculator.)

6. **[10 pts]** Given Matthew's tendency to get lonely sometimes, he decides to make some wholesome Zoom backgrounds that include his friends for the upcoming semester. Matthew has 100 friends and each Zoom background will contain exactly 3 friends (excluding Matthew himself). He also sets a rule that for any pair of two friends, they may only appear together in exactly one Zoom background. Given that Matthew can create as many backgrounds as he needs, is it possible for Matthew to include every distinct pair of two friends exactly once?

7. **[10 pts]** In preparation for her first online class at 9:00 AM, Kara has bought 509 pencils from her local pencil dealer. However, the pencil dealer is a little mean and has told Kara that he snuck one faulty pencil into the batch. Anyone who writes with the faulty pencil will lose all their fingers at exactly 8:59 AM. Kara wants to keep her fingers so she must discard the faulty pencil as soon as possible without directly writing with the pencils. Fortunately, Kara has 9 pencil-testing TA friends that would be willing to lose their fingers to save Kara's.

9:00 AM is quickly approaching! How can Kara organize the testing to identify the faulty pencil before her first class begins? Justify your answer.

Hint: Think Binary